

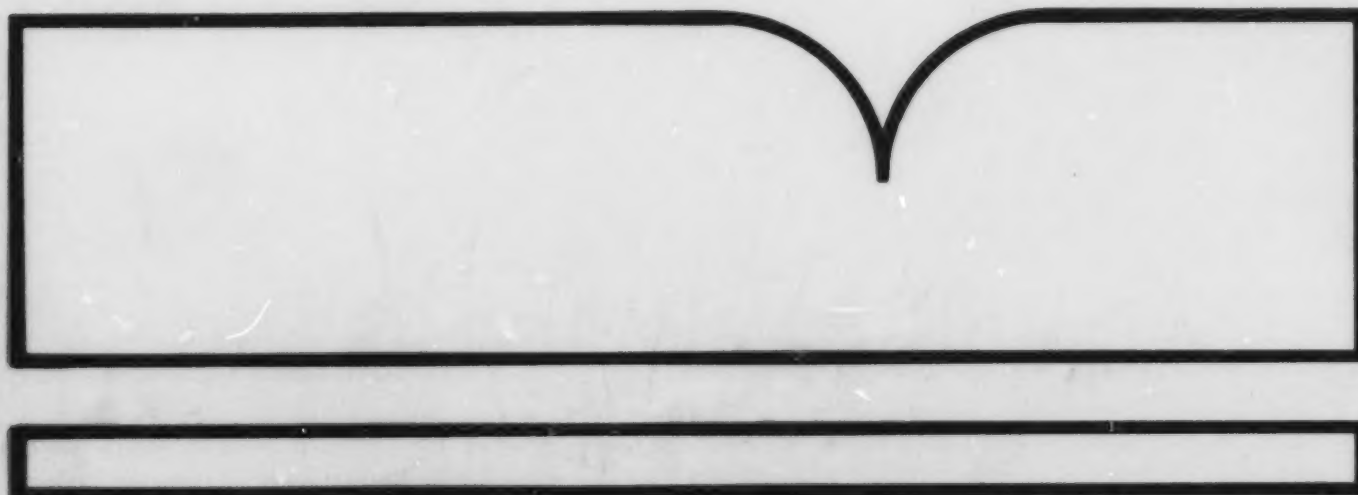
Time-Settlement Behavior of Processed Refuse
Part I: Mathematical Model for
Solid Waste Settlement

Northwestern Univ.
Evanston, IL

Prepared for

Municipal Environmental Research Lab.
Cincinnati, OH

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TIME-SETTLEMENT BEHAVIOR OF PROCESSED REFUSE

Part I - Mathematical Model for Solid Waste Settlement

by

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16. ABSTRACT The material investigated in this study is municipal refuse which has been processed in a hammer mill to produce a relatively fine-textured material in which the constituents are uniformly mixed. Such a material offers several advantages from an operational point of view, so that it is likely that the milling process will come into widespread use in the future. This material, in contrast to that of conventional sanitary landfill, lends itself to study and analysis because of its relative homogeneity. To model the time-settlement behavior of this material a generalized time-pore pressure dissipation relationship is combined with a rate process function which describes secondary consolidation to produce two simultaneous equations, one of which is non-linear. This model includes the effects of finite strains, biological and chemical activity, time variations in saturation, and large creep displacements. Functions are proposed to relate the instantaneous geometry and saturation to the instantaneous permeability, and the instantaneous geometry to the instantaneous compressibility. These are tentatively quantified using previously published data on the behavior of peat, which like the material studied is composed primarily of cellulose. The effects of the possible partial saturation on the stress distribution and ~~and~~ compressibility are discussed. Under conditions of total saturation, constant compressibility-permeability ratio, small strains, and no changes due to biochemical activity, the model reduces to the classical Terzaghi case.		
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FOREWORD

The Environmental Protection Agency was created because of increasing public and government concern about the dangers of pollution to the health and welfare of the American people. Noxious air, foul water, and spoiled land are tragic testimony to the deterioration of our natural environment. The complexity of that environment and the interplay between its components require a concentrated and integrated attack on the problem.

Research and development is that necessary first step in problem solution and it involves defining the problem, measuring its impact, and searching for solutions. The Municipal Environmental Research Laboratory develops new and improved technology and systems for the prevention, treatment, and management of wastewater and solid and hazardous waste pollutant discharges from municipal and community sources, for the preservation and treatment of public drinking water supplies, and to minimize the adverse economic, social, health, and aesthetic effects of pollution. This publication is one of the products of that research; a most vital communications link between the researcher and the user community.

This report evaluates a mathematical model of the time-settlement behavior of processed municipal solid waste (MSW). This investigation is timely since the hammer mill method for processing MSW is becoming increasingly popular.

Francis T. Mayo
Director
Municipal Environmental Research
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ABSTRACT

The primary constituent of most sanitary landfills is municipal refuse, which typically contains 60 percent waste paper with admixtures of glass, metals, dirt, garden and food wastes, and other discards of society. The time-settlement behavior of such a material cannot be described by classical consolidation theory because several of the assumptions of the theory are grossly violated. Biochemical processes which occur in such a material result in alteration of the solid phase and in gas generation, so that the saturation is not complete and varies with time. Also, finite strains and secondary consolidation must be accounted for, as well as continual variation in the permeability-compressibility ratio.

The material investigated in this study is municipal refuse which has been processed in a hammer mill to produce a relatively fine-textured material in which the constituents are uniformly mixed. Such a material offers several advantages from an operational point of view, so that it is likely that the milling process will come into widespread use in the future. This material, in contrast to that of conventional sanitary landfill, lends itself to study and analysis because of its relative homogeneity.

To model the time-settlement behavior of this material a generalized time-pore pressure dissipation relationship is combined with a rate process function which describes secondary consolidation to produce two simultaneous equations, one of which is non-linear. This model includes the effects of finite strains, biological and chemical activity, time variations in saturation, and large creep displacements. Functions are proposed to relate the instantaneous geometry and saturation to the instantaneous permeability, and the instantaneous geometry to the instantaneous compressibility. These are tentatively quantified using previously published data on the behavior of peat, which like the material studied is composed primarily of cellulose. The effects of the possible partial saturation on the stress distribution and compressibility are discussed.

The functions for the instantaneous permeability and compressibility are combined with the two simultaneous equations to reduce the number of variables in the model, and a one-dimensional finite difference solution is obtained for the two equations. The effects on the model response of the time variation of material properties, constant and time varying partial saturation, and biochemical activity are investigated to yield predicted time-settlement curves.

It is shown that under conditions of total saturation, constant compressibility-permeability ratio, small strains, and no changes due to biochemical activity, the model reduces to the classical Terzaghi case.

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ABBREVIATIONS AND SYMBOLS

$\bar{a}_v$	= average compressibility
A'	= area
A_s	= flow area
c_{a_v}	= average coefficient of compressibility ratio
C_m	= compressibility of unsaturated fluid
C_s	= shape constant
C_v	= Terzaghi consolidation coefficient
d	= measure of particle size
D	= creep deviator stress
e	= void ratio
e_f	= final void ratio
e'	= $e - e_1$
e_o	= initial void ratio
$\bar{f}$	= body force per unit mass
$\bar{g}$	= acceleration of gravity
G_s	= specific gravity of solids
h	= head of fluid phase
h_c	= Henry's solubility coefficient
i	= hydraulic gradient
k	= permeability
k_f	= final permeability
k_1, k_2	= permeability at specific void ratios

$\bar{k}$	$= \frac{\rho^f \rho^f g}{R^{sf} d^f} = \text{transmissibility}$
$\bar{k}_z$	$= \partial \bar{k} / \partial z$
m	$= \text{negative slope of log of strain rate- log of time curve}$
m_a	$= \text{air mass dissolved in a unit volume of water}$
$\bar{m}^\alpha$	$= \text{momentum charge in } \alpha \text{ phase due to internal interaction of other phases upon } \alpha$
n	$= \text{porosity of the solid matrix}$
$\bar{n}$	$= \text{normal vector to the surface}$
N	$= N' - N''$
N'	$= \text{power for void ratio-permeability relationship}$
N''	$= \text{constant for compressibility-void ratio function}$
N_s	$= \text{power of Brutsaert's transmissibility function}$
p^f	$= \text{internal fluid pressure}$
p^α	$= \text{internal pressure of } \alpha \text{ phase}$
P_a	$= \text{absolute atmospheric pressure}$
P_f	$= \text{pore pressure (absolute)}$
P_{go}	$= \text{initial gas pressure (absolute)}$
q	$= \text{surface tension of liquid}$
r_o	$= \text{radius of gas bubble}$
$\bar{r}$	$= \text{spatial axis vector}$
$\bar{r}^s$	$= \text{solid axis vector}$
R_f	$= \text{pore radius}$
$\bar{R}^{fB}$	$= \text{resistivity of other phases to fluid phase}$
S	$= \text{degree of saturation of the voids}$

S_e	= effective saturation
S_l	= volume surface
S^α	= total elemental volume of α phase - $S^{\alpha\alpha} + S^{\alpha B}$
$S^{\alpha B}$	= internal surface common between α phase and other phases
$S^{\alpha\alpha}$	= surface of elemental volume due to α phase
t	= time
$\bar{t}^\alpha$	= partial stress vector field
$\bar{t}^{\alpha\alpha}$	= stress tensor that represents the partial stress vector field
T	= dimension less unit time
$T_R = \frac{T}{T_s}$	= dimensionless Time Ratio
T_{sf}	= final structural time
u	= $\frac{u_f}{u_{f0}}$ = dimensionless fluid pressure
u_f	= excess fluid pressure
u_g	= excess gas pressure
U	= $\frac{e_o - e}{e_o - e_1}$
$\bar{v}$	= volumetric velocity relative to the fixed spatial system
$\bar{v}_{av}$	= average velocity
$\bar{v}^f$	= fluid velocity relative to spatial coordinates
$\bar{v}^s$	= solid phase velocity relative to the spatial coordinates
$\bar{v}^\alpha$	= α phase velocity relative to fixed spatial system
V	= bulk volume
V_d	= volume of dissolved gas

V_f	= volume of fluid
V_g	= volume of gas
V_v	= void volume
V^α	= volume occupied by the α phase
x^α	= phase axis
$\bar{x}$	= macroscopic volumetric axis
$\underline{\bar{x}}$	= fixed spatial axis
y^α	= phase axis
$\bar{y}$	= macroscopic volumetric axis
$\underline{\bar{y}}$	= fixed spatial axis
z^α	= phase axis
$\bar{z}$	= macroscopic volumetric axis
$Z = \frac{x}{H}$	= dimensionless distance
$\underline{\bar{z}}$	= fixed spatial axis
β	= soil creep parameter
γ	= unit weight
γ^α	= microscopic density of α phase
γ_w	= unit weight of water
$\delta \text{ vol}$	= microscopic volume
ϵ	= strain
$\bar{p}^\alpha$	= partial stress that complements the internal pressure of α
η	= coefficient of absolute viscosity
θ^α	= fraction of volume occupied by the α phase
θ^f	= fluid fraction of elemental volume
$\lambda = 1 - U = \frac{e - e_1}{e_0 - e_1}$	

u	$= \frac{e_o - e}{e_o - e_f}$
ρ^α	= bulk density of α phase
ρ^f	= bulk density of the fluid phase
$\bar{\sigma}$	= effective stress
$\bar{\sigma}_T$	= total stress
$\bar{\sigma}_n^\alpha$	= normal component of $\bar{m}^\alpha$
τ	= viscous resistance
$\bar{\tau}^\alpha$	= tangential component of $\bar{m}^\alpha$
ϕ_c	= soil creep parameter
α	= parameter for stress equation
γ^α	= the amount of α phase generated -- the time rate of increase of mass of α per unit volume
∇	= del operator
Δ	= operator denoting an infinitesimal quantity
$\frac{d}{dt}$	= total material time derivative operator
$\frac{\partial}{\partial x_i}$	= partial space derivative operator
$\frac{\partial}{\partial t}$	= partial time derivative operator

CHAPTER I

INTRODUCTION

Solid waste disposal is fast becoming one of man's more pressing environmental problems, particularly in the United States with its high per capita consumption of disposable goods. Each year over 3½ billion tons of refuse is produced in the United States. About 5 percent or 200 million tons of this total output is municipal refuse, while the remaining 95 percent is a result of mining, agricultural and other industrial activities (Thadani, 1971). Because of its composition and origin, the municipal refuse must be collected and disposed of while much of the industrial waste such as mine tailings is left at or near its source.

The problems faced by those responsible for refuse collection and disposal are: increasing amounts of materials, increasing labor cost, increasing haul distances and a lack of disposal sites. At the same time many new ideas to save mankind from his trash are being proposed. The U. S. Bureau of Mines has conducted research on reclaiming materials in refuse (Kenahun, 1971), while others propose such solutions as making building bricks from waste (Gershowitz, 1971). Many people advocate recycling of materials such as glass and paper. Although ideally it offers the ultimate solution, recycling can only be effective when the economic demand for the material is great. Any solution of the refuse disposal problem must be economically more favorable than the present methods. Examples of unsuccessful attempts at recycling waste products can be found in the closing of most composting plants, because the resultant product, a rich, bulky fertilizer, cannot be given away. Although some recycling

operations are locally quite successful, there will always be considerable volumes of municipal refuse that must be disposed of.

A landfill's basic function is to provide an economic final disposal site for the unwanted waste products of the society it serves. Sanitary landfills are attempts to reduce the health and fire hazards associated with open dumps. Additional improvements in the public safety and aesthetic qualities of the sanitary landfill may be produced by milling the refuse prior to disposal.

This study of milled refuse is undertaken to provide an understanding of some of the time-settlement characteristics of the material while its use is still uncommon. The information gained will help in planning future landfills to provide the most economical and ecologically best solution.

The objective of this study is to develop a mathematical model of the time-settlement behavior of milled refuse and examine its response in relation to variations in geometry, and bio-chemical activity. The model developed will include the effects of large deflections, biological and chemical activity, and the large amount of creep that takes place. The process and effects of milling will be discussed in Chapter III.

CHAPTER II

GENERAL COMPOSITION AND BEHAVIOR OF REFUSE

On a given day, in a given city, the refuse produced and collected may be very different than that collected on another day. However, during the entire year, the average composition of the refuse will be very similar for most United States cities. Upon placement of the material in a landfill, many chemical and biological changes will take place, and will change the characteristics of the materials. An analogy between the behavior of refuse and soil materials, particularly peat, can be made and will be used to model the material.

Landfill Composition

The primary constituent of most landfills is municipal refuse, which contains a wide variety of items. Cinders, lumber, unwanted cars, household garbage and construction waste are a few of the ingredients, yet over one-half of all this type of refuse is composed of cellulose. An analysis of municipal refuse in the Los Angeles area yielded the following percentages by weight: 15 percent non-combustible, 15 percent garbage, and 70 percent combustible, with the greatest potential for increase being in the combustible waste (Astrojet-General Corporation, 1965). The non-combustible waste contains no cellulose, while most of the solids in the other two groups are cellulose.

Garbage consists of the waste from food handling, preparation and serving and is mostly vegetable matter. A considerable part of the vegetable matter is water, but the solid material is primarily cellulose. The combustible waste is composed of paper (54 percent), grass and leaves (10 per-

cent), wood (27 percent), rags (2 percent), leather and rubber and synthetics (2 percent)(Astrojet-General Corporation, 1965). The paper and rags are nearly completely cellulose and wood is about two-thirds cellulose (the rest is lignin and resins). The solid components of the grass and leaves are about the same as wood (Institute of Resources Research of Japan, 1968). Thus the cellulose content by percent of the total weight is: paper, wood and rags, 57 percent; grass and leaves, 4 percent; garbage, 6 percent; or a total of 67 percent. This overall percentage is substantiated by the fact that when refuse is ashed at high temperatures (400°C), 60 to 70 percent of the oven dry weight (at 105°C) is lost. Since nearly two-thirds of the total waste is cellulose and nearly all of the biodegradable waste is cellulose, it is therefore quite likely that the landfill's activity will be similar to the behavior of cellulose under the same conditions -- modified, however, by the presence of materials that are chemically active or will supply nutrients for microbial growth.

The non-combustible rubbish consists of metals, glass, stone, ceramics, ashes and dirt. Of these items, only the metals possess the potential for breaking down under the conditions likely to be encountered in a landfill. Making up 7 percent of the refuse by weight, the metals and their decomposition products are of interest on a volume as well as on a pollution basis. The metals may react with the free oxygen or the organic acids produced during anaerobic digestion.

Biological and Chemical Decomposition

The biological and chemical activity of the refuse will influence its behavior in several ways. First, the assimilation of the organic components

by microorganisms for energy and growth will reduce the total amounts of solids slightly. Second, the biological activity, as well as secondary reactions between the end products such as organic acids and alcohols and the undegradable materials, will affect the characteristics of the remaining material. The third effect is a change in the permeability and flow patterns in the material as a result of the gases released. The extent, type, and rate of these interactions will depend upon the type of organisms that can exist in the material.

General Classifications of Biological Activities

The type of organisms, rate of reaction and completeness of the reaction can be generally classified into one of four categories. These groups are described by the organism's source of oxygen and the general temperature range.

The two major temperature groupings are the mesophilic and thermophilic ranges (Loehf, 1966). During the transition between the two temperatures ranges, the activity of one group will decrease while that of the other will increase. No organisms of importance in decomposition are thought to exist above the thermophilic range, but temperatures as high as 92.3°C are reported in a California study (Ralph Stone, Inc., 1968). The amount of activity will vary considerably between the two groups, the thermophilic activity being greater, and within either range (mesophilic 20 to 40°C , thermophilic 45 to 60°C) the microbial metabolism rate will double for about every 10°C increase in temperature.

The oxygen source of the organisms determines the type of process that takes place and its rate to a greater degree than does the temperature.

The aerobic assimilation of organic materials by bacteria is generally a complete process, with water, carbon dioxide and cell protoplasm being the primary end products. In the presence of sufficient food, water, and oxygen, the process is nutrient controlled. Because the reactions go to completion, the process is one of high energy yield with 484 to 647 kilo-calories of heat being released for every gram molecule of glucose oxidized. Constant aeration is needed to supply organisms with sufficient oxygen for this process because of the high rate of oxygen consumption. However, it is likely that in a typical landfill only a small volume near the surface will receive sufficient oxygen to remain aerobic for any extended period, and that after a brief time most of the biological activity will become anaerobic (Eckenfelder, 1966; Fair, Geyer and Okam, 1968; Gotaas, 1956; Rode, 1962).

While the mass is aerobic, humification of some of the material may take place. In this process some of the decomposition products are partially oxidized, polymerized, consolidated and combined. These products can in turn decompose only at a very slow rate and tend to accumulate. This is the same process that produces the humus in the topsoil (Rode, 1962).

If little gaseous oxygen is present, the slow anaerobic digestion will proceed. In this process anaerobic and facultative microorganisms obtain oxygen by shifting it from one complex molecule to another or by removing it from certain mineral compounds. That this is not a complete reaction is indicated by the relatively low energy yield of 26 kilo-calories per gram molecule of glucose, and the high energy end products such as methane and organic acids. Unless external heat energy is supplied or excellent insulation exists, the temperatures will usually fall into the mesophilic range. This is a two step process wherein the complex molecules are broken down to

less complex ones which are then fermented into gases. Initially, Saprophytic bacteria secrete enzymes which convert the complex molecules into less complex soluble organic compounds by hydrolysis. These and other organisms then oxidize the hydrolyzed products into simpler organic compounds such as fatty acids. These acids are then fermented to methane, carbon dioxide and small quantities of more complex organic gases in the second step by methane bacteria. The overall rate of this process is governed by the kinetics of the slowest step which is thought to be the methane fermentation (Carpenter, Vamvakias, and Gellman, 1968; Jeris and McCarty, 1965; Lawrence and McCarty, 1966; Milhulits and Malina, 1968; Rode, 1962; Schroeder and Busch, 1966; Technical Practice Committee, 1966).

The chemical and biological activity of the landfill will continually cause changes in the physical and chemical structure of the material, depending upon pretreatment of refuse such as shredding and/or compaction. The activity, composition and certain mechanical parameters of the system are dependent on time and one another. Presently, however, the processes that take place are not fully understood and their time rate has not been established.

The biological activity will be anaerobic for the majority of the time; however, the brief initial aerobic period may have a disproportionate influence on the consolidation characteristics of the mass. In particular, if the material is shredded prior to landfill, considerable surface area will be exposed to atmospheric oxygen, which will remain in the material after it is in place. Until this initial gaseous oxygen is consumed, the active organisms will be aerobic and produce considerable amounts of carbon dioxide, which may collect in the larger pores of the material and greatly reduce

its permeability. After the initial aerobic period, the mass will become anaerobic. The type or group of anaerobic organisms that are active will depend on the conditions present at any given time and will likely vary during the lifetime of the landfill. As was stated before, the rate at which the process proceeds depends upon the kinetics of the slowest step. Aside from this, the speed can also be controlled by the products which may be toxic to organisms. Analyses of anaerobic digestion of pulp and paper waste indicate that the rate of cellulose decomposition is generally controlled by the amount of nitrogen available; in a landfill it is likely that the nitrogen and the toxins will control the process.

The relationship between time and biological activity for the mass will show erratic changes that are due to the continually changing conditions. The dominance of a particular type of organism over others will depend upon the internal factors such as availability of water and nutrient, and accumulation of end products as well as the external temperature and rainfall. Any given organism is likely to alter a favorable environment so that other organisms will increase in importance. It has been found that in anaerobic digestors used for sewage treatment, the temperature of the process will remain in the mesophilic range because of low energy yield. However, several feet of landfill or topsoil should provide sufficient insulation so that the loss of heat will be small enough to maintain the thermophilic range. Although the exact rates have not been established, the relative rates of decomposition can be estimated if the temperature and environmental conditions are known.

Mechanical Considerations

The purely mechanical aspects of refuse settlement involves the rearrangement of the solids and expulsion of fluids from the mass, and is similar to the consolidation of a soil. However, refuse settlement will differ greatly from that of the classical material contemplated by the Terzaghi theory (Terzaghi and Peck, 1967). To begin with, most of the assumptions of the classical theory are invalid and, in addition, the process is not dominated by the action of excess pore fluid dissipation. Terzaghi assumes that only infinitesimal movements occur, the relative movement of the solids and fluid can be ignored, permeability and compressibility are a constant ratio, the material is saturated and that only the flow of pore fluid is involved in the consolidation. None of these assumptions are valid for refuse. Large finite deflections will occur, the flow of fluid will depend upon the relative motion of the fluid and solid particles, permeability and compressibility will vary greatly, saturation will vary greatly, and a large part of the settlement will be a result of creep. The term creep is defined to be the movements of the material which are not controlled by the dissipation of excess fluid pressure. In addition to these differences, there is the possibility of the changes in pore fluid due to the biological and chemical activity. These problems will be dealt with in the following chapters. Since a large percentage of the substances present are composed of organic compounds similar to those found in peat, the behavior of the two materials should also be similar.

CHAPTER III

MILLED REFUSE

The material investigated in this study is common urban refuse that has been shredded in a hammer mill to change several of its characteristics. As a result of this processing, the material becomes a homogeneous mass which will behave in a different manner than conventional refuse. The common refuse landfill is a heterogeneous mixture of most of the discards of the society it serves. In the United States a landfill may contain old car bodies, refrigerators, bed springs, as well as papers, garbage and other waste. Such a landfill defies mathematical description and the mechanical characteristics of any part of it would be of little value in predicting the overall behavior.

At the present time, the milling of refuse is growing in popularity in this country because of the many desirable effects that result from this process. In the milling process, large untreatable items such as refrigerators and bed springs are first removed, and then the materials are dropped through a series of rotating steel hammers which shred the paper, pulverize wood and brittle objects such as glass, and eject most large pieces of metal. The particular milling installation observed for this study is located in and operated by the City of Madison, Wisconsin.

The physical act of milling produces several desirable results in the material. First and most important for this study is "that it is homogeneous, and its individual components such as newspapers and plastic bottles, are not recognizable. Milled refuse has an appearance of oversized confetti or torn newsprint." (Reinhardt, 1971.) Second, higher in-place

densities are obtainable with the milled refuse than conventional (1000 lb/cu yd vs 850 lb/cu yd) and when the volume required for cover dirt is considered, only 570 lb/cu yd of conventional refuse can be placed in a landfill. Third, the putrescible matter is finely divided and distributed nearly uniformly through the paper's bulk, resulting in a material that will not support rodents and insects, and eliminating the need for daily cover dirt. Fourth, the decomposition tends to occur faster than in the unpulverized material, resulting in a faster stabilization of the materials. Another advantage is that because of the smaller size and intertanglement of the "confetti," there is very little wind blown paper. Also, because of more uniform and smaller voids, the material is less susceptible to fire.

As with any process, there are certain disadvantages. First, the cost of disposal from pickup to placement is greater for the milled than unmilled refuse (\$2.80 versus \$2.25 per ton)(Reinhardt, 1971). However, the savings in the cost of acquiring additional disposal space should help offset the added cost. Second, the milled refuse is somewhat bulkier initially, although later it compacts to a greater density than the unmilled refuse. Third, there is a high initial capital cost.

The characteristics of the Madison waste along with those given in Chapter II are shown in Table 3-1. There are several items which differ significantly between the two analyses; these variations may be seasonal or a result of collection practice. The low amount of paper products in the Madison refuse could be because the city picks up bundled paper for recycling. On the other hand, the large volume of garden waste present is likely due to the seasonal leaf fall.

Analysis was made of the particle size after milling by Reinhardt (1971),

TABLE 3-1

COMPOSITION OF URBAN REFUSE

Component	Percent by Weight		
	Reinhardt (1971)	Astrojet General (1965)	De Marco, <u>et al</u> (1969)
Food Waste	15.3	15.0	22.3
Garden Waste	13.8	10.0	4.4
Paper Products	42.4	54.0	47.6
Plastics, Rubber, Leather	1.8	2.0	2.8
Textiles	1.6	2.0	2.2
Wood	1.1	2.0	1.4
Metals	6.7	7.0	8.3
Glass and Ceramics	10.1	3.0	6.8
Rocks, Dirt, Ashes, etc.	7.2	5.0	4.4

Reinhardt's data are from Madison, Wisconsin.

Astrojet General's data are from Los Angeles, California.

De Marco et al's data are from Johnson City and Memphis, Tennessee; Ogden, Utah; and Alexandria, Virginia.

and these data are shown in Figure 3-1. The refuse was simply shaken through a series of sieves, and the amount retained on each was weighed. There is, as would be expected, a relationship between the grate size through which the milled material passes and the particle size. The term "particle size" refers to the two larger dimensions of a piece of material. For example, a "particle" 1" x 2" x 1/8" would be retained on the next sieve smaller than one inch, and the thickness would have no bearing on the "particle size" determination, although it would have a major influence on the void geometry. Thus the "particle size" has no direct bearing on the permeability.

The refuse is shown prior to milling in Figure 3-2a. Large cardboard boxes, soda cans and a Christmas tree are easily identified in the picture. After being placed in the receiving hopper, it is carried on a conveyor belt to the top of the mill, as shown in Figure 3-2b. The finished product, as well as the hammer mill, are shown in Figure 3-3. The uniformity of size and lack of bulky items should be noted. The Madison landfill site for the milled refuse is shown in Figure 3-4a. Again, the uniformity should be pointed out. Also, the ease with which the tractor is supported is unique to this type of refuse. At Madison, the milled refuse "has been used successfully to build access roads to desired dumping areas. This characteristic facilitates sanitary landfill operations during inclement weather" (Reinhardt, 1971). This material has been left uncovered for over a year with no problems. An idea of the relative size of the material can be obtained from Figure 3-4b, which shows a common plastic ballpoint pen along with a plastic bottle and the shredded paper.

Although the initial capital cost is high, and the direct placement cost is greater than conventional landfill methods, the process of milling

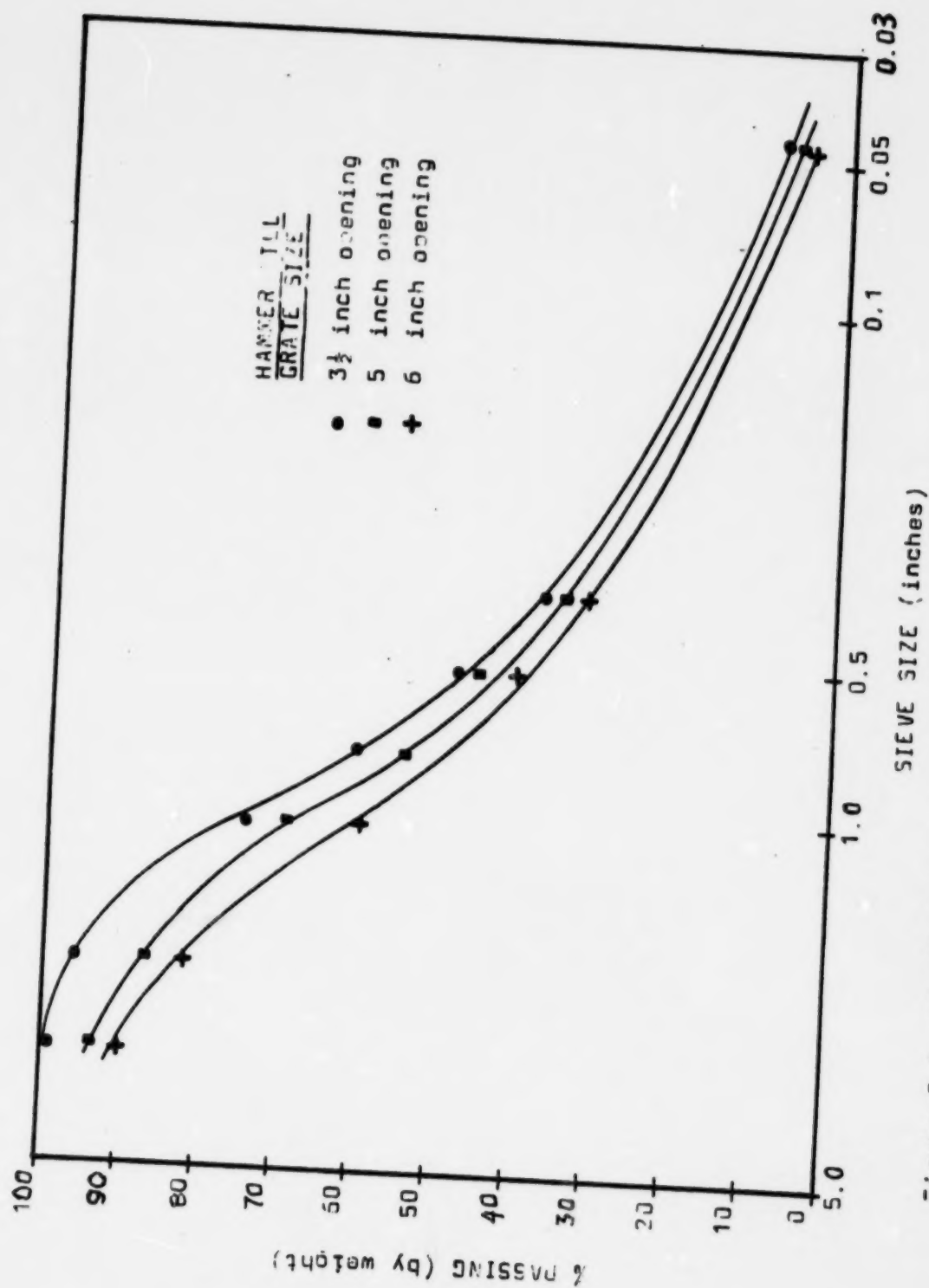


Figure 3-1: "Particle" size analysis of milled Madison, Wisconsin refuse (after Reinhardt, 1971).



a) Unmilled refuse in feed hopper



b) Conveyor to mill

Figure 3-2





Figure 3-3 : Milled refuse and hammer mill

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a) Active milled refuse site in Madison



b) Milled refuse in place

Figure 3-4

offers many long run savings and advantages. In the future, a much higher premium will be placed upon the space required for a landfill, and thus the space savings of the milling process are very attractive. Also, the homogeneity introduced produces fairly uniform settlements of the surface materials. As a result of the fast stabilization and more uniform settlements, a milled refuse site can be returned to a useful economic function in a shorter time than the conventional site. With the awakening of man to his effect on his environment and the advantages offered by the milling process, it is likely that the use of the process will continue to increase.

CHAPTER IV

CONSOLIDATION THEORY

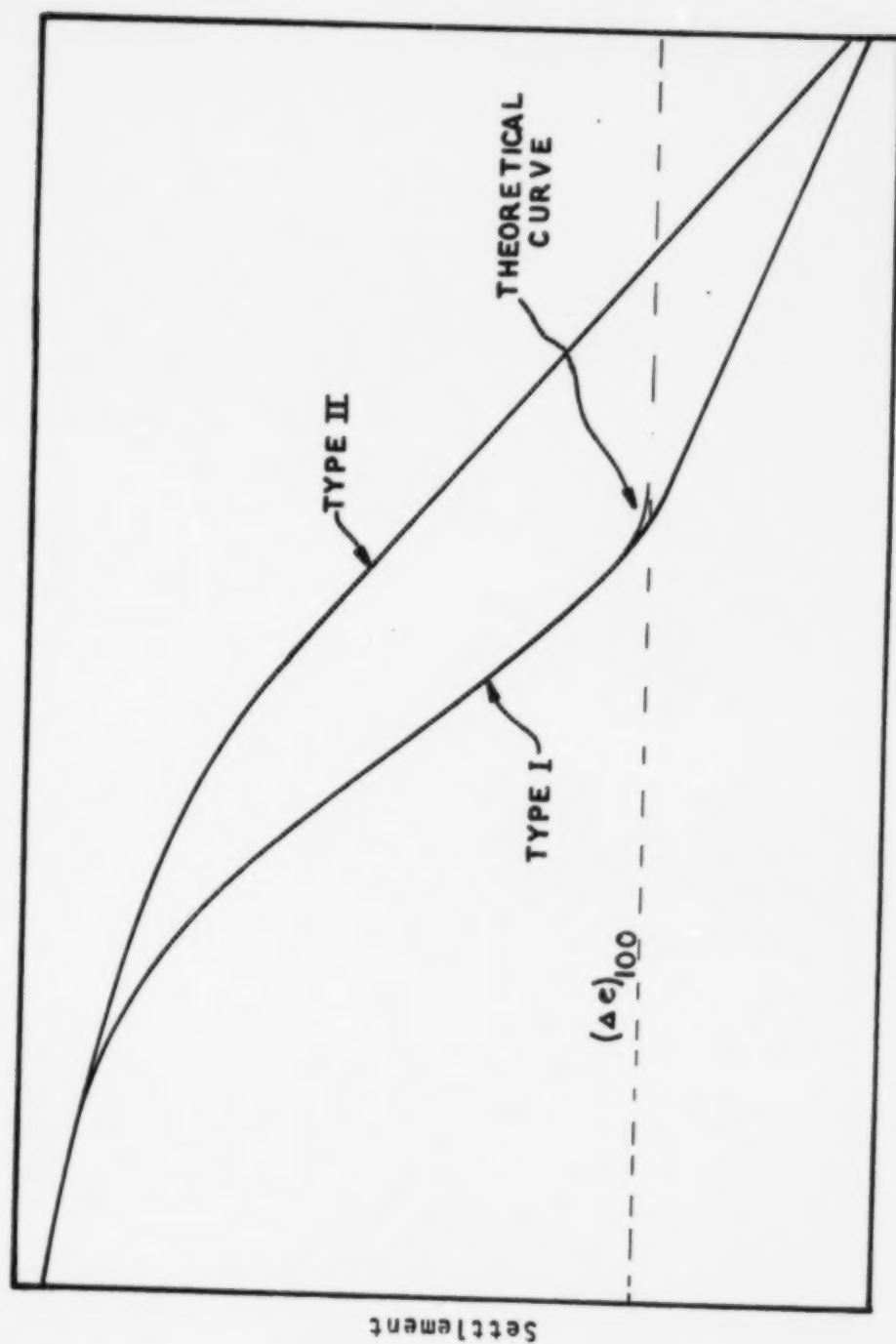
The consolidation, or volume reduction, of a sanitary landfill results from a combination of two distinct components: the mechanical response of the particles and pore fluids to loading and the chemical and biological decomposition. The mechanical component of consolidation is classically associated with two processes, one being pore pressure dissipation and the other simultaneous creep which persists, however, after the pore pressure dissipation is complete. The mechanics of the reduction of the pore pressures is relatively well understood and theory is available by which the time rate of this process can be estimated. The mechanics of the creep process are not well understood and the mathematical analysis is, so far, dependent on the success of curve fitting methods. Biological and chemical activity in a landfill will produce some volume change directly, but their greatest effects are in their influence on the mechanical processes through the generation of gases and the alteration of the structure of the solid matrix. These activities will produce relatively large volumes of gas that will alter the makeup of the pore fluid and greatly change the permeability of the landfill; in addition to this there will be less important structural changes in the solid matter. The permeability and pore fluid changes will alter the rate of pore pressure dissipation, while the structural variation may influence the creep rate. A mathematical model for these three processes and their interactions are developed in this chapter.

Classical consolidation theory deals with an inert saturated material that is undergoing small deformations and no creep. The material considered

in this study is not necessarily inert, or saturated, and is undergoing considerable deformations as well as creep. Approximately two-thirds, by weight, of the refuse is composed of some form of cellulose matter, while the other third is of relatively inert solid material; it is likely that the behavior of the milled refuse will resemble in many respects that of some peats, with the added effect of relatively high rates of biological and chemical activity. Examination of the studies of the properties and response of peat, therefore, promises to give some insight into the behavior of the refuse.

Wahls (1962) discusses the two general types of consolidation curves for soils: those that follow Terzaghi's theory, and those that do not. This may be an oversimplification, but most soils and peats that greatly differ from the classical behavior tend to have similar deflection versus log time graphs. Wahls' curves are shown in Figure 4-1.

Type I follows the theoretical Terzaghi curve nearly to 100 percent consolidation and in this phase the response is dominated by the process of dissipation of fluid pressures. After this dissipation, the response is due to the creep mechanism and the curve void ratio versus time becomes nearly linear. Type II, on the other hand, shows a pressure dissipation type of response for a short time, but its response is soon dominated by creep. The difference between the two types of behavior is in the relative importance of the two mechanisms in the total settlement. The departure from the classical type of behavior may result from several different sources. If an inorganic clay, which when loaded normally would behave as Terzaghi's theory predicts, is given only one-fourth or one-eighth of the standard load increment, the resulting deformation-time relationship will



Log. Time

Figure 4-1: Typical curves for consolidation (after Wahls 1962)

be of Type II. In such a case the creep process will dominate throughout most of the test, giving the non-classical curve. Other soils, such as organic clays, soils with considerable organic content, and peats give curves that lie between the two curves of Figure 4-1. The actual response curves of the other soils depend upon the type of loading and the strength and composition of their solid matrix.

Lea (1958), Schroeder and Wilson (1962), Barden and Berry (1962), and others, discuss the variations between different types of peats and those of soil and peats and their effect on the consolidation of the materials. The pore pressure dissipation process is in general the same for both soil and peat, except that the strains are quite large in the latter. The secondary consolidation, or creep, mechanism differs between peat and soil as well as the various types of peat themselves. Although the creep processes in clays are not fully understood, in general they are due to one of the bound water physical chemical processes (Wahls, 1962). Since the structure of peat can vary from the highly colloidal amorphous-granular to the fibrous, it is not surprising that great differences in creep behavior have been observed. As a consequence, several different creep models have been proposed. The fine grained peats are thought to creep for the same reasons as does clay; the intrusion of bound water layers, bond jumping and adjustment of shear forces. Fibrous peat, however, is thought to have two levels of drainage; the first follows the general pore pressure dissipation theory, while the second follows the creep theory. Despite the differences between the cause of creep in the different types of peat, several studies (Bardon and Berry, 1966; Schroeder and Wilson, 1962; Hanrahan, 1954) have found that the creep in both the

fibrous and amorphous-granular types of peat follows, in general, the linear settlement versus log time relationships, as shown in Figure 4-2. The two curves for the English fibrous peat in the figure illustrate the effect of the size of the load increment upon the behavior of the material. Curve C is for a load increase of only one-fourth and the curve appears to be of Wahls' Type II, while for D the ratio is unity and the material behaves more nearly as the Terzaghi theory would predict.

Consolidation of peat and soils differ in several other respects. Laboratory strains of 20 to 30 percent and as high as 50 percent are common for peat compared to several percent for most clays. Hanrahan (1954) found the permeability of a peat to decrease by a factor of 50,000 during consolidation, while Lea (1958) found the coefficient of consolidation to change by a factor of 100. Under these conditions, the basic assumptions of small deformations, constant permeability, and compressibility, as well as the static form of Darcy's Law used in the Terzaghi theory, are not valid for peat.

Thus, the problem is to model a material that is undergoing large strains, has changing properties, may or may not be saturated, has considerable creep, and might undergo biological and chemical processes. Several studies have included one or more of the variations from classical theory; however, none have made a reasonable combination that will fit the problem at hand. Davis and Raymond (1965) and Raymond (1969) considered the effects of small variations in the compressibility and permeability as did Poskitt (1969). Barden (1965a, 1968, 1969) considers the simultaneous effect of primary and secondary consolidation for saturated clays with small deformations. He says that the classical Terzaghi theory oversimpli-

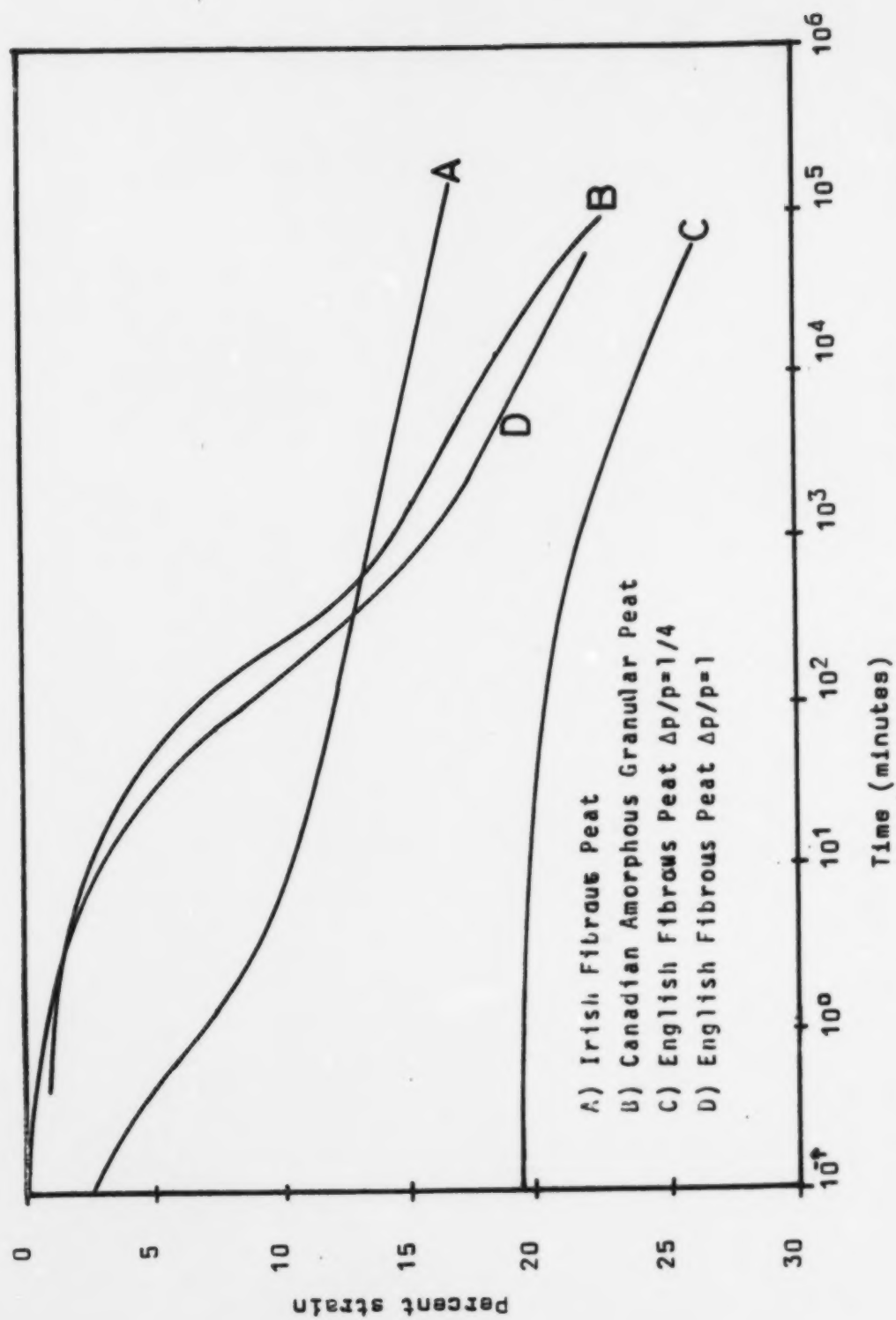


Figure 4-2: Strain versus time curves for peats (after Barden and Berry, 1968)

fies the process of consolidation because the consolidation does not stop when the pore pressure is zero, and the rate of dissipation of the pore pressure is greater in the mid-plane than the theory predicts. Barden says primary and secondary consolidation occur simultaneously and he introduced a term for the viscous resistance of the soil into his basic pressure versus void ratio relationship. Gibson et al (1967) developed a model based on large deformations and showed that decreases in compressibility during consolidation increased the time required to reach a given degree of consolidation. The effect of saturation on consolidation has been considered by several workers. Yoshimi and Osterberg (1963) found that for low values of saturation only structural deformations (creep) need be considered. Barden (1965b) investigated the entire saturation range (0 to 100 percent) while Toriyama and Sawada (1968) considered consolidation of clay where the saturation was greater than 85 percent.

A general relationship for the effect of saturation on the pore pressure phase of consolidation of a material must be based on an analysis of a three phase model; that is, it must consider a solid, water, and gas system. Barden (1965b) developed such a relationship for a clay soil. Assuming a two phase pore fluid, flowing one-dimensionally through the soil, under isothermal conditions, and obeying Henry's Law, he gives the continuity equations for the air and water phases respectively as follows:

$$\frac{\partial}{\partial x} [v_g \rho_g + m_g v_w] = \frac{\partial}{\partial t} [m_g S n + \rho_g (1 - S) n] \quad (4-1)$$

$$\frac{\partial}{\partial x} [v_w \rho_w] = \frac{\partial}{\partial t} [n S \rho_w] \quad (4-2)$$

where

- m_a = air mass dissolved in a unit volume of water
- n = porosity of the solid matrix
- s = degree of saturation of the voids
- v_a = discharge velocity of the air
- v_w = discharge velocity of the water
- t = time
- x = direction of flow
- ρ_a = density of air
- ρ_w = density of water.

If the water is assumed to be incompressible and Boyle's and Darcy's laws to apply, two simultaneous equations in terms of four independent variables are obtained. The needed two additional relationships are not available. Barden, however, solved this problem by considering five different ranges of saturation and the variations in the pore fluid inherent for each as the boundary conditions.

The first of the five different cases is one of a very dry soil, where all the water present is bound to the solid matrix and the air phase is completely interconnected. The effect of the primary consolidation (air-flow) for this case is negligible, as was shown by Yoshimi and Osterberg (1963), and the mechanics involved is one of primarily creep, or structural deformation. The second case is like the first, except that there is a greater saturation and the mathematics are more involved. The settlement is still primarily due to creep. Barden's third case is one of transition where the air and water transmissibilities are nearly equal and both fluids

flow at about the same rates. The solution of this case requires an understanding of the behavior of the air-water interface which is not presently available. Fortunately, this case is a transient one, lasting only a very short time, and not being able to solve it is of little consequence. The fourth and fifth categories of Barden are of the greatest interest from the primary consolidation standpoint. For these cases the air present can exist in two states: one is as free bubbles that move with the pore water as it moves through the solid matrix and the second is as trapped air that occupies dead pore space. The two pore fluid concept becomes quite cumbersome due to the sorption and movements of the air and the resulting changes in the water transmissibility of the material. A solution can be obtained if a single homogeneous fluid pore fluid with non-changing compressibility is assumed in place of the two fluid model. Barden uses this single fluid approach for the last two cases and finds that the consolidation behavior is governed primarily by the variation in the permeability and secondarily by the compressibility.

Toriyama and Sawada (1968) also investigated an unsaturated soil, but restricted their studies to the high saturation areas (Barden cases 4 and 5). Using the single pore fluid concept and more general relationships for the saturation and coefficient of volume change than did Barden, they found that for the high range of saturation the rate of consolidation increases as the degree of saturation prior to the test, the initial pore pressure, and the compressibility of the material increases. Although both Barden and Toriyama and Sawada present good treatments of the effect of saturation of clays, they do not consider finite deformations or the effects of creep.

The development of the general model for pore pressure dissipation

requires that consideration be given to both pore pressure and creep processes that occur at the same time. Barden (1965a, 1969) and Poskitt (1971) both deal with this problem and use the general relationship

$$c \frac{\partial}{\partial x} [F(k, e, \gamma_w, x, \dots)] \frac{\partial u_f}{\partial x} = \frac{\partial \mu}{\partial t} \quad (4-3)$$

where

- c = constant
- e = void ratio
- k = permeability
- u_f = pressure in pore fluid
- x = spatial coordinate in the flow direction
- γ_w = unit weight of water
- $\mu = \frac{e_o - e}{e_o - e_f}$
- e_o = initial void ratio
- e_f = final void ratio.

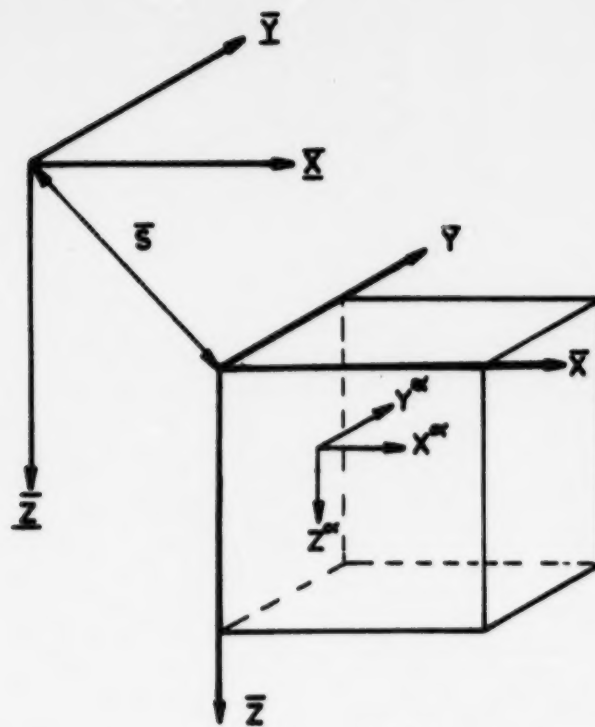
Barden's work emphasizes the non-linear influence of the creep on the material behavior, while Poskitt deals more with the changing permeability and compressibility. Poskitt's model includes changing soil properties and creep effects, and employs the method of perturbation to arrive at a solution. As a result, he studies a "mildly non-linear consolidation process," which indirectly includes the effects of finite strain. Barden's method of solution to the problem employs two simultaneous equations, one dealing only with the pore pressure process and the other with creep.

Because of the freedom that results from the simultaneous equation approach as outlined in the 1965 paper, it is used in this study. The different parts of the model must now be developed.

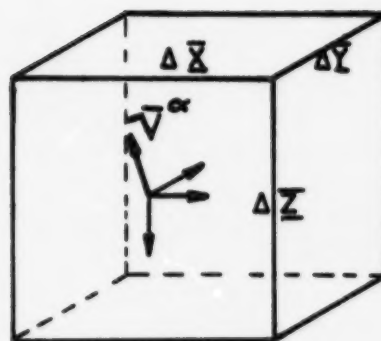
In addition to the mechanical problems, the materials are unstable, and will undergo biological and chemical changes. The exact nature of these processes is not understood, but they will produce several effects on the mechanical action. First, any chemical change in the materials will somewhat alter the mechanical properties of the materials. Second, small amounts of gas will greatly affect the transmissibility as well as the pore fluid makeup. Also, small amounts of liquids such as organic acids, alcohols, fats or water will be generated by these changes.

Excess Fluid Pressure Dissipation

The general model for the pore pressure dissipation of a material requires that large deflections, multiphase pore fluid, and phase changes be considered. Gibson et al (1967) develop a large-deflection equation for a two phase system (soil and water) using the relative velocity between the phases, while Raats and Klute (1968a) use multiple coordinate systems for their analysis of multi-phase materials. The development here will utilize multiple coordinate systems because this approach lends itself to a more general analysis. Consider the system shown in Figure 4-3a with the three sets of coordinate axes. The set $\bar{X}$, $\bar{Y}$, $\bar{Z}$ are rectangular cartesian coordinates that represent a fixed spatial system that does not change with time. The two other sets are Lagrangian: that is, they change with time relative to the spatial system and possibly each other, but remain fixed with respect to two points. At any time t , the macroscopic volumetric



(a)



(b)

Figure 4-3: Co-ordinate systems

system $\bar{x}, \bar{y}, \bar{z}$ is located at position $\bar{S}$ relative to the fixed spatial set. Also each phase, α , has associated with it the coordinates $x^\alpha, y^\alpha, z^\alpha$, which may be related to the volumetric system. The volumetric as well as the phase-related systems move with respect to the spatial system at the velocities $\bar{v}$ and $\bar{v}^\alpha$.

The necessity for the multiple coordinate system arises from the fact that some of the variables are space- as well as time-dependent. When using such an analysis, care must be taken to assure that the reference to a system is correct and that the correct transformations are made when moving from one system to another. Long (1963) defines the material time derivative of a function $\mathcal{F}$ to be "the rate at which $\mathcal{F}$ is changing as we follow a moving element of the continuum." This is related to the partial derivative expressing the rate of change of $\mathcal{F}$ at a given point by a convection term which depends on the velocity of the element. The relationship given by Long is

$$\begin{aligned} \frac{d\mathcal{F}}{dt} &= \frac{\partial \mathcal{F}}{\partial t} + v_x \frac{\partial \mathcal{F}}{\partial x} + v_y \frac{\partial \mathcal{F}}{\partial y} + v_z \frac{\partial \mathcal{F}}{\partial z} \\ &= \frac{\partial \mathcal{F}}{\partial t} + v_i \frac{\partial \mathcal{F}}{\partial x_i} \end{aligned} \tag{4-4}$$

where

$$\frac{d\mathcal{F}}{dt} = \text{total material time derivative}$$

$$\frac{\partial \mathcal{F}}{\partial t} = \text{partial time derivative } (x_i \text{ constant})$$

$$v_i = \text{velocity of the } i^{\text{th}} \text{ element}$$

$$\frac{\partial f}{\partial x_i} = \text{partial space derivative}$$

By use of equation 4-4, the change in a function f relative to the phase, or material, coordinates x^α , y^α , z^α can be related to the spatial axes $\bar{x}$, $\bar{y}$, $\bar{z}$ as long as proper partial derivatives exist.

The macroscopic bulk density ρ^α of the α phase is the mass of this phase per unit bulk volume. Mathematically, this bulk density can be expressed as

$$\rho^\alpha = \frac{\int_V \rho_p^\alpha dV}{V} \quad (4-5)$$

where ρ_p^α is the microscopic or point density of the α phase and V is bulk volume. The point density of liquids and gases may vary locally, such as in the case of water between two clay particles. The average point density ρ_a^α is given by

$$\rho_a^\alpha = \frac{\int_V \rho_p^\alpha dV}{V^\alpha} \quad (4-6)$$

where V^α is the volume occupied by the α phase. The fraction θ^α of the volume occupied by the α phase is

$$\theta^\alpha = \frac{\rho^\alpha}{\rho_a^\alpha} \quad (4-7)$$

At any given point the mass flux of the α phase is

$$\rho^\alpha \bar{v}^\alpha = \rho^\alpha[\bar{s}, t] \bar{v}^\alpha[\bar{s}, t] \quad (4-8)$$

where $\bar{v}^\alpha$ is the discharge velocity of the α phase, $\bar{S}$ is the position vector and t is time. Both the bulk density and velocity are field quantities; that is, functions of time and space. Consider an elemental volume $\Delta \bar{x} \Delta \bar{y} \Delta \bar{z}$, shown in Figure 4-3b, of the spatial system where the only variable is time. The total flux of the α phase through such a system is

$$\begin{aligned} \Delta \bar{x} \Delta \bar{y} \Delta \bar{z} [\nabla(\rho^\alpha, \bar{v}^\alpha)] &= \Delta \bar{x} \Delta \bar{y} \Delta \bar{z} \left[\frac{\partial}{\partial \bar{x}} (\rho^\alpha \bar{v}^\alpha) \cdot i_{\bar{x}} \right. \\ &\quad \left. + \frac{\partial}{\partial \bar{y}} (\rho^\alpha \bar{v}^\alpha) \cdot i_{\bar{y}} + \frac{\partial}{\partial \bar{z}} (\rho^\alpha \bar{v}^\alpha) \cdot i_{\bar{z}} \right] \end{aligned} \quad (4-9)$$

in which $i_{\bar{x}}$, $i_{\bar{y}}$, and $i_{\bar{z}}$ represent the unit vectors in the spatial system. The spatial time rate of change is equal to the generation rate of the α phase, ψ^α , less the outflow; or

$$\Delta \bar{x} \Delta \bar{y} \Delta \bar{z} \frac{\partial \rho^\alpha}{\partial t} = [\psi^\alpha - \nabla \cdot (\rho^\alpha \bar{v}^\alpha)] \Delta \bar{x} \Delta \bar{y} \Delta \bar{z} \quad (4-10)$$

Assuming that there is a solid and one or more fluid phases, the model will be developed using the solid phase as the working reference system. For a fluid phase f , the spatial time rate of change is, from equation 4-10.

$$\frac{\partial \rho^f}{\partial t} = -\nabla \cdot (\rho^f \bar{v}^f) + \psi^f \quad (4-11)$$

Now the total material time derivative of this fluid phase relative to the solid phase is given by equation 4-4. It is

$$\frac{d\rho^f}{ds_t} = \frac{\partial \rho^f}{\partial t} + \bar{v}^s \cdot \nabla \rho^f \quad (4-12)$$

where the s denotes the solid phase and $\bar{v}^s$ is the solid phase velocity relative to the spatial coordinates. In equations 4-11 and 4-12, the only part not described in terms of spatial coordinates is the left hand side of equation 4-12. Combining these two equations, we get

$$\frac{d\rho^f}{ds_t} = -\nabla(\rho^f \cdot \bar{v}^f) + \bar{v}^s \cdot \nabla \rho^f + \psi^f \quad (4-13)$$

By addition and subtraction of the term $\rho^f \nabla \cdot \bar{v}^s$ equation 4-13 can be rewritten

$$\frac{d\rho^f}{ds_t} = -\rho^f \nabla \cdot \bar{v}^s + \psi^f - \nabla[\rho^f(\bar{v}^f - \bar{v}^s)] \quad (4-14)$$

Now the first term of the right hand side will be put into a more convenient form by evaluating the divergence of the solid phase velocity. The balance of mass of the solid phase, in spatial terms, may be written

$$\frac{\partial \rho^s}{\partial t} = -\nabla(\rho^s \cdot \bar{v}^s) + \psi^s \quad (4-15)$$

From equation 4-4, the total material time derivative of the solid phase relative to itself is

$$\frac{d\rho^s}{ds_t} = \frac{\partial \rho^s}{\partial t} + \bar{v}^s \cdot \nabla \rho^s \quad (4-16)$$

Substituting equation 4-15 into equation 4-16 and expanding yields

$$\frac{d\rho^s}{dt} = -\rho^s \nabla \bar{v}^s - \bar{v}^s \nabla \rho^s + \bar{v}^s \nabla \rho^s + \psi^s. \quad (4-17)$$

Assuming that no solid phase change exists, we find the divergence of the solid phase to be

$$\nabla \bar{v}^s = \frac{-1}{\rho^s} \frac{d\rho^s}{d^s t} \quad (4-18)$$

Substituting equation 4-18 into equation 4-14 yields

$$\frac{d\rho^f}{d^s t} - \frac{\rho^f}{\rho^s} \frac{d\rho^s}{d^s t} + \nabla [\rho^f (\bar{v}^f - \bar{v}^s)] - \psi^f = 0. \quad (4-19)$$

If it is assumed that the fluid phase changes are uniform throughout the mass then ψ^f is independent of location; and if the third term is evaluated relative to the solid coordinates rather than the spatial, equation 4-19 becomes

$$\rho^s \frac{d(\rho^f/\rho^s)}{d^s t} + \frac{\partial}{\partial \bar{r}^s} [\rho^f (\bar{v}^f - \bar{v}^s)] \frac{\partial \bar{r}^s}{\partial \bar{r}} - \psi^f = 0 \quad (4-20)$$

where $\bar{r}^s$ is referred to the solid frame of reference and $\bar{r}$ to the spatial. The term $\partial \bar{r}^s / \partial \bar{r}$ is the deformation gradient of the solid phase relative to the spatial coordinates. Equation 4-20 allows the boundary conditions to be expressed in terms of the solid phase, as needed for the consolidation problem. Development of this general relationship may be found in various works such as Flugge (1960), Raats and Klute (1968a), Soo (1967), and others.

In order to evaluate this equation, the fluid flux relative to the solid phase and the deformation gradient must be known. Raats and Klute (1968b) develop the needed expressions for the relative flux term, based upon the balance of momentum of the system as outlined in the following paragraphs. In a multi-phase system there are five possible sources of momentum changes. The first source is due to convection, or the outflow and inflow of the material. The mass belonging to the α phase that crosses any surface element in a unit time is

$$\rho^\alpha \bar{v}^\alpha \cdot \bar{n} \quad (4-21)$$

where $\bar{n}$ is the normal vector of the surface. At any point on the surface the momentum change is

$$(\rho^\alpha \bar{v}^\alpha \cdot \bar{n}) \bar{v}^\alpha. \quad (4-22)$$

The total change due to convection is thus

$$-\int_{S_v} (\rho^\alpha \bar{v}^\alpha \cdot \bar{n}) \bar{v}^\alpha dS_v \quad (4-23)$$

where s_v is the volume surface, and the negative sign results from taking the positive velocity as being outward.

The second source of momentum change is due to the effect of all phases outside the unit volume upon the α phase within the elemental volume. This action is represented by a field of partial stress vectors, $\bar{t}^\alpha$, and the change in momentum due to this is

$$\int_{S_V} \bar{t}^\alpha dS_V \quad (4-24)$$

The body forces acting due to external force fields upon the α phase are the third source of momentum change. The resultant of this effect is

$$\int_V \rho^\alpha \bar{f}^\alpha dV \quad (4-25)$$

where $\bar{f}^\alpha$ is the body force per unit mass.

The momentum also changes as a result of the internal interaction of the different phases. If the effect of this interaction is denoted as $\bar{m}^\alpha$ then the total gain in momentum for the α phase is given by

$$\int_V \bar{m}^\alpha dV. \quad (4-26)$$

The last source of momentum change is due to any generation of the α phase within the volume. If ψ^α is the time rate of increase of mass of α per unit volume, then the change in momentum due to this is

$$\int_V \psi^\alpha \bar{v}^\alpha dV. \quad (4-27)$$

The sum of these five effects is the total time rate of change of momentum and the general expression is

$$\begin{aligned} \frac{\partial}{\partial t} \left\{ \int_V \rho^\alpha \bar{v}^\alpha dV \right\} = & - \int_{S_V} (\rho^\alpha \bar{v}^\alpha \cdot \bar{n}) \bar{v}^\alpha dS_V + \int_{S_V} \bar{t}^\alpha dS_V \\ & + \int_V \rho^\alpha \bar{f}^\alpha dV + \int_V \bar{m}^\alpha dV + \int_V \psi^\alpha \bar{v}^\alpha dV. \end{aligned} \quad (4-28)$$

The first term on the right hand side of this equation can be changed from a surface to volume integral using Green's theorem, or

$$\int_{S_V} (\rho^\alpha \bar{v}^\alpha \cdot \bar{n}) \bar{v}^\alpha dS_V = \int_V \nabla [(\rho^\alpha \bar{v}^\alpha) \bar{v}^\alpha] dV \quad (4-29)$$

The field of partial stress vectors, $\bar{t}^\alpha$, that act upon the volume can be represented by a stress tensor, $\bar{\bar{t}}^\alpha$, and the two are related by

$$\bar{t}^\alpha = \bar{\bar{t}}^\alpha \cdot \bar{n} \quad (4-30)$$

where $\bar{n}$ is the unit normal vector.

Substituting equation 4-30 into the second term of the right hand side of equation 4-28 and using Green's theorem yields

$$\int_{S_V} \bar{t}^\alpha dS_V = \int_{S_V} \bar{\bar{t}}^\alpha \cdot \bar{n} dS_V = \int_V \nabla \bar{\bar{t}}^\alpha dV \quad (4-31)$$

If the phases are separate and have surfaces within the volume that can interact with each other, then the rate of momentum change, which is a force, that occurs within the volume can be broken into normal and tangential components and is expressed as

$$\bar{m}^\alpha = \bar{\sigma}_n^\alpha + \bar{\tau}^\alpha \quad (4-32)$$

where $\bar{\sigma}_n^\alpha$ is the normal component and $\bar{\tau}^\alpha$ is the tangential component.

Substituting equations 4-29, 4-31, and 4-32 into 4-28 yields

$$\begin{aligned} \frac{\partial}{\partial t} \left\{ \int_V \rho^\alpha \bar{v}^\alpha dV \right\} &= - \int_V \nabla [(\rho^\alpha \bar{v}^\alpha) \bar{v}^\alpha] dV + \int_V \nabla \bar{t}^\alpha dV \\ &+ \int_V \rho^\alpha \bar{f}^\alpha dV + \int_V (\bar{\sigma}_n^\alpha + \bar{\tau}^\alpha) dV + \int_V \psi^\alpha \bar{v}^\alpha dV. \end{aligned} \quad (4-33)$$

Now since the volume is time independent, the order of differentiation and integration of the left hand side can be changed, and taking the derivative of both sides with respect to the volume yields the differential form of the spatial balance of momentum for the α phase or

$$\begin{aligned} \frac{\partial}{\partial t} (\rho^\alpha \bar{v}^\alpha) &= - [(\rho^\alpha \bar{v}^\alpha) \bar{v}^\alpha] + \nabla \bar{t}^\alpha + \rho^\alpha \bar{f}^\alpha \\ &+ \bar{\sigma}_n^\alpha + \bar{\tau}^\alpha + \psi^\alpha \bar{v}^\alpha \end{aligned} \quad (4-34)$$

Equation 4-34 can be rewritten in terms of material coordinates. The transformation equation (4-4) for the α phase relative to itself is

$$\frac{d\rho^\alpha \bar{v}^\alpha}{d^\alpha t} = \frac{\partial \rho^\alpha \bar{v}^\alpha}{\partial t} + \bar{v}^\alpha + \nabla \bar{v}^\alpha \rho^\alpha. \quad (4-35)$$

Substituting 4-35 into 4-34 and adding and subtracting yields

$$\begin{aligned} \frac{d\rho^\alpha \bar{v}^\alpha}{d^\alpha t} + \rho^\alpha \bar{v}^\alpha \nabla \bar{v}^\alpha - \psi^\alpha \bar{v}^\alpha &= \\ \nabla \bar{t}^\alpha + \rho^\alpha \bar{f}^\alpha + \bar{\sigma}_n^\alpha + \bar{\tau}^\alpha \end{aligned} \quad (4-36)$$

As with equation 4-14, the divergence of $\bar{v}^\alpha$ must be evaluated. Using the balance of mass and the connecting equations for the α phase relative to itself, the divergence is found to be

$$\nabla \bar{v}^\alpha = -\frac{1}{\rho^\alpha} \frac{d\rho^\alpha}{d\alpha_t} + \frac{\psi^\alpha}{\rho^\alpha} \quad (4-37)$$

Substituting 4-37 into 4-36 yields

$$\begin{aligned} \frac{d\rho^\alpha \bar{v}^\alpha}{d\alpha_t} - \frac{\rho^\alpha \bar{v}^\alpha}{\rho^\alpha} \frac{d\rho^\alpha}{d\alpha_t} + \bar{v}^\alpha \psi^\alpha - \bar{v}^\alpha \psi^\alpha \\ = \nabla \bar{t}^\alpha + \rho^\alpha \bar{f}^\alpha + \bar{\sigma}_n^\alpha + \bar{\tau}^\alpha \end{aligned} \quad (4-38)$$

or

$$\rho^\alpha \frac{d\bar{v}^\alpha}{d\alpha_t} = \nabla \bar{t}^\alpha + \rho^\alpha \bar{f}^\alpha + \bar{\sigma}_n^\alpha + \bar{\tau}^\alpha \quad (4-39)$$

This is the balance of forces upon the α phase. The left hand side represents the inertial force of the phase, while the right hand side contains the contact forces, body forces and forces due to the other phases.

The surface of the elemental volume that belongs to the α phase is denoted as $S^{\alpha\alpha}$. Then the general elemental surface and the α phase surface are related by

$$dS^{\alpha\alpha} = \theta^\alpha dS_V \quad (4-40)$$

where θ^α is defined by equation 4-7. The different surfaces are shown in Figure 4-4a. The partial stress vectors $\bar{t}^\alpha$ represents the action of all external phases upon the surface $S^{\alpha\alpha}$, and if it is assumed that their action is equal to the internal pressure of the α phase, p^α , plus an extra partial stress $\bar{\bar{p}}^\alpha$, the relationship between $\bar{t}^\alpha$, p^α , and $\bar{\bar{p}}^\alpha$ is

$$\int_{S_V} \bar{t}^\alpha dS_V = \int_{S^{\alpha\alpha}} p^\alpha \cdot \bar{n} dS^{\alpha\alpha} + \int_{S^{\alpha\alpha}} \bar{\bar{p}}^\alpha \cdot \bar{n} dS^{\alpha\alpha} \quad (4-41)$$

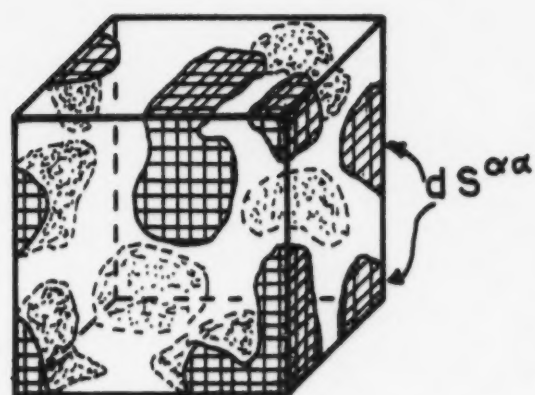
The internal pressure of the α phase also acts upon all internal surfaces of contact $S^{\alpha\beta}$ of the α phase with the other phases (Figure 4-4b) and is equal to the normal component of the internal momentum transfer. The result of this is

$$\int_V \bar{\sigma}_n^\alpha dV = \int_{S^{\alpha\beta}} p^\alpha \cdot \bar{n} dS^{\alpha\beta} \quad (4-42)$$

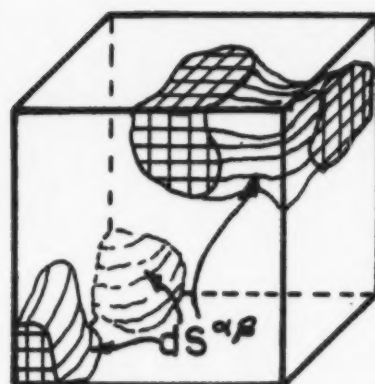
Adding equations 4-41 and 4-42 will give the effect of $\bar{t}^\alpha$ and $\bar{\sigma}_n^\alpha$, or

$$\begin{aligned} \int_{S_V} \bar{t}^\alpha dS_V + \int_V \bar{\sigma}_n^\alpha dV &= \int_{S^{\alpha\alpha}} p^\alpha \cdot \bar{n} dS^{\alpha\alpha} \\ &+ \int_{S^{\alpha\beta}} p^\alpha \cdot \bar{n} dS^{\alpha\beta} + \int_{S^{\alpha\alpha}} \bar{\bar{p}}^\alpha \cdot \bar{n} dS^{\alpha\alpha} \end{aligned} \quad (4-43)$$

The total surface of the α phase in the elemental volume, S^α , is the sum of the two partial surfaces or $S^{\alpha\alpha} + S^{\alpha\beta}$, so that the first two terms on the right hand side can be combined into one. Doing this, and



(a)



(b)

Figure 4-4: Elemental surfaces

using equations 4-30 and 4-40 yields from equation 4-43,

$$\int_{S_V} \bar{t}^\alpha \cdot \bar{n} dS + \int_V \bar{\sigma}_n^\alpha dV = \int_{S_V} -p^\alpha \cdot \bar{n} dS + \int_{S_V} \theta^\alpha \bar{\rho}^\alpha \cdot \bar{n} dS \quad (4-45)$$

By use of Green's theorem, the equation becomes

$$\begin{aligned} \int_V \nabla \bar{t}^\alpha dV + \int_V \bar{\sigma}_n^\alpha dV = \\ \int_V -\theta^\alpha \nabla p^\alpha dV + \int_V \nabla (\theta^\alpha \bar{\rho}^\alpha) \cdot \bar{n} dS. \end{aligned} \quad (4-46)$$

This can be differentiated to give

$$\nabla \bar{t}^\alpha + \bar{\sigma}_n^\alpha = \theta^\alpha \nabla p^\alpha + \nabla (\theta^\alpha \bar{\rho}^\alpha). \quad (4-47)$$

Substituting equation 4-47 into 4-39 yields

$$\begin{aligned} \rho^\alpha \frac{d\bar{v}^\alpha}{dt} = -\theta^\alpha \nabla p^\alpha \\ + \nabla (\theta^\alpha \bar{\rho}^\alpha) + \rho^\alpha \bar{f}^\alpha + \bar{\tau}^\alpha \end{aligned} \quad (4-48)$$

If it is assumed that the α phase is a fluid and that its flow is laminar, then the inertial forces can be considered negligible (Polubarinova-Koching, 1962). Assuming also that the only body force of consequence is gravity, equation 4-48 becomes

$$-\theta^f \nabla p^f + \nabla (\theta^f \bar{\rho}^f) + \rho^f \bar{g} + \bar{\tau}^f = 0 \quad (4-49)$$

where $\bar{g}$ represents the acceleration of gravity. If it is assumed that the tangential force γ^f and the divergence of the extra partial stresses, $\nabla(\theta^f \bar{p}^f)$, depend upon the velocity of the fluid relative to the other phases and time, equation 4-49 becomes

$$-\theta^f \nabla p^f + \rho^f \bar{g} = F[(\bar{v}^f - \bar{v}^\theta), \bar{R}^{f\theta}, t] \quad (4-50)$$

where θ represents the other phases and $\bar{R}^{f\theta}$ the resistivity to relative motion between the f phase and the others. If it is assumed that the function F is a linear and isotropic function of the velocity difference and there exists a two phase system (solid and fluid), equation 4-50 becomes

$$-\theta^f \nabla p^f + \rho^f \bar{g} = R^{fs}(\bar{v}^f - \bar{v}^s) \quad (4-51)$$

Multiplying both sides by ρ^f gives

$$\rho^f(\bar{v}^f - \bar{v}^s) = -\frac{\rho^f \rho^f}{R^{fs}} \left(\frac{1}{\rho_a^f} \nabla p^f - \bar{g} \right) \quad (4-52)$$

where ρ_a^f is defined by equation 4-6. This is the needed relationship for equation 4-20; we have the relative flux expressed in terms of measurable quantities. However, the right hand side must be written in terms of material (solid) coordinates. Using the same notation as before, equation 4-52 becomes

$$\rho^f(\bar{v}^f - \bar{v}^s) = -\frac{\rho^f \rho^f}{R^{fs}} \left(\frac{1}{\rho_a^f} \frac{\partial p^f}{\partial \bar{r}^s} \frac{\partial \bar{r}^s}{\partial \bar{r}^f} - \bar{g} \right). \quad (4-53)$$

Equation 4-53 is a form of Darcy's law in which the velocity of the fluid is measured relative to the solid phase. Zaslavsky (1964) says:

Darcy's formula and the hydraulic conductivity $\bar{K}$ are meaningless unless q (flow) is actually measured relative to the soil particles. Otherwise, extremely different values of $\bar{K}$ can be found for the same soil under the same flow conditions and gradients only because the soil is translated at different velocities with respect to discharge-measuring pipes.

If we consider these observations, equation 4-53 is the proper form of Darcy's law, particularly when large deformations can occur.

The pressure and head of the fluid are related by

$$h = \frac{p^f}{\rho_a^f g} \quad (4-54)$$

where h is the head in units of length, and g is the magnitude of the gravitational acceleration. If h_g is the gravitational head, then

$$\frac{\partial h_g}{\partial \bar{r}} = g \quad (4-55)$$

Relating 4-55 to solid coordinates and combining it and 4-54 with equation 4-53, we have

$$\frac{\rho^f}{\rho_a^f} (\bar{v}^f - \bar{v}^s) = - \frac{\rho^f \rho_a^f g}{R^{sf} d^f} \frac{\partial \bar{r}_s}{\partial \bar{r}} \left[\frac{\partial (h + h_g)}{\partial \bar{r}_s} \right] \quad (4-56)$$

If

$$\bar{K} = \frac{\rho^f \rho_a^f g}{R^{sf} t^f} \quad (4-57)$$

then

$$\theta^f (\bar{v}^f - \bar{v}^s) = \bar{k} \frac{\partial \bar{r}_s}{\partial \bar{r}} \left[\frac{\partial (h + h_{gl})}{\partial \bar{r}_s} \right] \quad (4-58)$$

Equation 4-58 is a general form of Darcy's law that is applicable to both saturated and unsaturated media that may or may not be undergoing movements of their solid phases.

Substitution of 4-58 into 4-20 yields

$$\rho^s \frac{d(\rho^f/\rho^s)}{d^s t} - \frac{\partial}{\partial \bar{r}_s} \left[d^f \bar{k} \frac{\partial \bar{r}_s}{\partial \bar{r}} \left(\frac{\partial (h + h_{gl})}{\partial \bar{r}_s} \right) \right] \frac{\partial \bar{r}_s}{\partial \bar{r}} - \psi^f = 0. \quad (4-59)$$

Equation 4-59 is the generalized form of the consolidation equation. If the medium is completely saturated

$$\theta^f = \frac{\rho^f}{\rho_a} = n = \frac{e}{1+e} \quad (4-60)$$

where n is the porosity and e the void ratio. If the water is incompressible, then the first term in equation 4-59 is

$$\begin{aligned} \rho^s \frac{d(\rho^f/\rho^s)}{d^s t} &= - \frac{\rho^f}{\rho^s} \frac{d\rho^s}{d^s t} \\ &= \frac{\rho^f}{\left(\frac{g \gamma_w}{1+e} \right)} \frac{d \left(\frac{g \gamma_w}{1+e} \right)}{d^s t} = \frac{\rho^f}{1+e} \frac{de}{dt} \end{aligned} \quad (4-61)$$

where γ_w is the microscopic unit weight of water. In the Terzaghi equation it is assumed that the amount of deformation of the solid phase is so small that there is no change in the spatial volume occupied by the solids. This reduces the solid and spatial coordinates to a single system, and the solid velocity becomes zero while the deformation gradient becomes unity. If no fluid is produced and equations 4-60 and 4-61 are introduced into 4-59, the resulting equation is

$$\frac{\rho^f}{1+e} \frac{\partial e}{\partial t} = -\nabla(\rho^f \bar{v}^f) \quad (4-62)$$

which is Terzaghi's equation.

One Dimensional Case

In the generalized consolidation equation, 4-59, the dimensional aspects of the problem are considered in the deformation gradient $\partial \bar{r}_s / \partial \bar{r}$. For the two and three dimensional cases, this is a complex expression; however, for the one-dimensional case Raats and Klute (1969) give

$$\frac{\partial \bar{r}_s}{\partial \bar{r}} = \begin{vmatrix} \partial \bar{r}_{s_x} / \partial r_x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \rho^s / \rho_o^s & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad (4-63)$$

The bulk densities can be expressed in terms of volume or void ratio:

$$\frac{\partial \bar{r}^s}{\partial \bar{r}} = \frac{\rho^s}{\rho_o^s} = \frac{V_o}{V} = \frac{1+e_o}{1+e} \quad (4-64)$$

If the material is assumed to be homogeneous and to be at equilibrium

prior to the load increase, substituting 4-64 into 4-59 and changing the head to pressure yields

$$\rho^s \frac{\partial(\rho^f/\rho^s)}{\partial t} - \frac{\rho^s}{\rho_o^s} \frac{\partial}{\partial \bar{r}_s} \left(\frac{\bar{K} \rho_a^s}{\gamma_w} \frac{\rho^s}{\rho_o^s} \frac{\partial u_f}{\partial \bar{r}_s} \right) - \psi^f = 0 \quad (4-65)$$

or letting $\bar{r}_s = x$,

$$\rho^s \frac{\partial(\rho^f/\rho^s)}{\partial t} - \frac{\rho^s}{\rho_o^s} \frac{\partial}{\partial x} \left(\frac{\bar{K} \rho_a^s}{\gamma_w} \frac{\rho^s}{\rho_o^s} \frac{\partial u_f}{\partial x} \right) - \psi^f = 0 \quad (4-66)$$

where u_f is the excess fluid pressure. This is the general one-dimensional equation for the dissipation of pore pressure.

Creep

In contrast to the detailed analysis available for the pore fluid controlled consolidation, the creep or structural deformation is just beginning to be understood. Until the analogy between soil and colloid behavior was made, analysis of creep in soils was based entirely upon curve fitting methods. It has since been established that fine grained soils and amorphous-granular peats are in fact colloidal materials and that the surface forces, rather than gravitational, determine its behavior; and that the energy relationships used in colloidal theory are valid for these materials.

Some possible causes of creep are thought to be "gradual readjustment of frictional forces, plastic deformation of the adsorbed water, jumping of clay bonds, and viscous structural reorientation caused by shear stresses." (Wahls, 1962.) Mitchell (1964) and Christensen and Wu (1964) both proposed that the rate process theory be used to model the process of

secondary consolidation. Using this theory, they related the creep to the effective stress, temperature, soil structure, rate of strain, and cohesion of the soils. Wu et al (1966), using rate process theory, give an expression to model the secondary consolidation in the form of

$$\frac{de}{dt} = \beta \sinh(\phi_c \tau) \quad (4-67)$$

where de/dt is the strain rate, β and ϕ_c are soil parameters, and τ is the viscous resistance.

Mitchell, Campanella, and Singh (1968) discuss the validity of the rate process theory when applied to soils. They point out that most previous writers have assumed that the theory is a valid explanation of the process of secondary consolidation and proceed to use mathematical methods to substantiate this assumption. Their work establishes the following points for creep in clay:

1. It is temperature dependent.
2. The "activation energy" decreases with an increasing deviator stress.
3. The strain rate depends on the deviator stress.
4. The creep rate decreases with time.

However, they state that "quantities (constants) determined at any one time may not be the same at some other time."

From their study the form of the rate process theory which may be used to model creep in soils is

$$\frac{de}{dt} = \epsilon'_0 \left(\frac{t_1}{t}\right)^m \sinh(C_1 D) \quad (4-68)$$

where

t, t_1 = time

D = viscous resistance

ϵ'_0 = strain rate at time t_1 , $D = 0$

C_1 = constant

m = negative slope of log of strain rate versus log of time curve

Singh and Mitchell (1968) say that in the past it has often been thought that after some initial period of time the creep continues for a very considerable period at a constant rate and that the strain versus log time relationship is linear. They point out that previous studies indicate that although steady-state creep may exist for a brief period of time, it appears that the creep rate decreases with time, and that the logarithm of strain rate versus logarithm of time is a linear relationship for some clays.

Walker (1969b) says that Singh and Mitchell's equation "appears to represent an oversimplification of the macroscopic response of clay structure to an applied stress system," and that testing conditions will have considerable influence on the values of ϵ'_0 , m and C_1 . In another paper, Walker (1969a) states that associated with the creep is a pore pressure increase which reflects additional volumetric deformations, but that these follow a straight log time curve. Also, he observes that the soil behaves macroscopically in creep as a viscous material with a viscosity that increases with time.

Barden (1968) applied the same theory he used in a previous paper (1965a) to a wider range of materials than he or any others had done before.

In this paper he demonstrates that a macro-micro pore structure mechanism of creep which may occur in fibrous peat and some clays can be modeled by the same theory as the bond jumping mechanism. He points out that peat presents a very complex consolidation problem, due to the moving boundaries and the large permeability decrease, but that a good approximation to its creep behavior can be obtained from the micro-macro pore theory. In a later paper, Barden (1969) uses the rate process model of Wu et al (1966) in the simultaneous equation approach (1965a) to the overall consolidation of peat and clay, and found that this creep model yielded as satisfactory results as his previous model (1968).

Creep of peat has been studied less than that of clay, but test results indicate that the process in peat is similar in appearance to the rate process theory for clay. Hanrahan (1954) found, using remolded peat, that under increasing loads, the amount of primary consolidation increases, the duration of the primary phase is nearly equal for the different loads, and after the primary consolidation is completed, the creep proceeds at the same rate in all tests. The results of these tests are shown in Figure 4-5. As can be seen from the results, the secondary, or structural dominated phase approaches a linear settlement versus log time plot.

Adams (1965) found that after most of the initial excess pore pressures (one inch of water pressure remained), the consolidation continued at a much slower rate, nearly linear with the logarithms of time. Barden and Berry (1968) note the same time-settlement relationship, and suggest that the rate process relation equation, 4-67, be used for creep. The difficulty in using equation 4-67 or 4-68 is in the evaluation of different constants, and determination of just how "constant" they are.

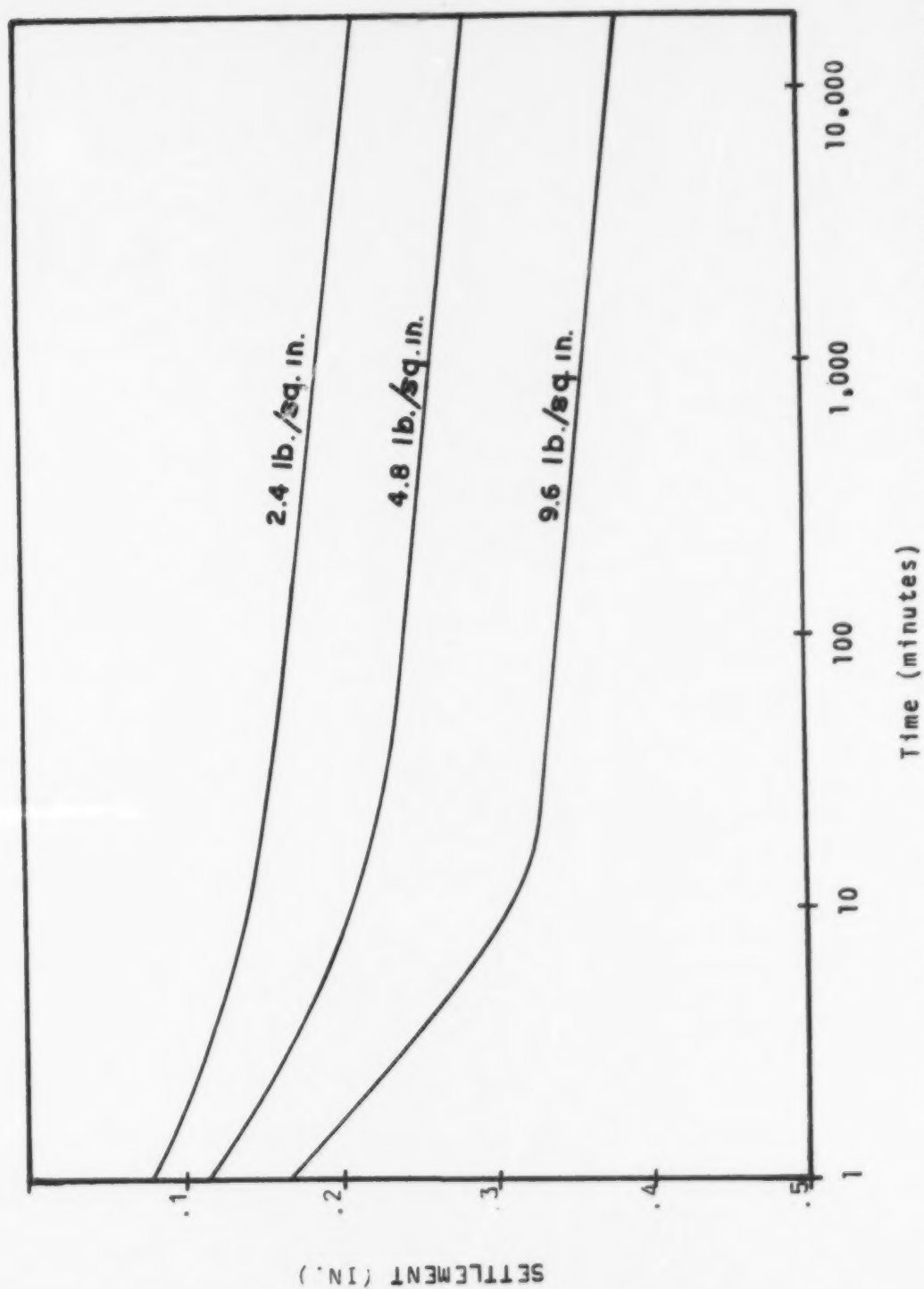


Figure 4-5: Effect of varying load upon remolded peat(after Hanrahan 1954)

For the consolidation of peat, a comparison between the field and laboratory time-settlement behavior is of importance because of the large amounts of secondary compression and the relatively low gradients. Goodman and Lee (1962) found very good agreement between laboratory and field settlements based solely upon the classical theory. They point out that the secondary settlements are small compared to what might be expected. Forrest and MacFarlane (1969) found good agreement between their field and laboratory tests also. However, they found a considerable amount of creep and produced a time-settlement graph which lies somewhat between the two curves of Wahls. It is unlikely that the peats used in these two studies were at all similar and thus it is not surprising that there are some differences. The importance of both studies is that they indicate that laboratory and field behavior for some peats are similar. Thus, with the creep of both peat and clay being well described by equation 4-67, and the similarity between the make up of peat and milled refuse, it is assumed that this relationship will hold for the milled refuse which is contemplated in the present study.

Combined Model

The total consolidation model will be developed by first expanding the pore pressure dissipation (equation 4-66) function, and then combining it with the creep function (equation 4-67). The bulk densities of the solid and fluid phases as well as the average density of the fluid phase can be expressed in more conventional terms as

$$\begin{aligned}\rho^s &= \frac{G_s \gamma_w / \bar{g}}{1+e} \\ \rho^f &= \frac{S e \gamma_w / \bar{g}}{1+e} \\ d^f &= S \gamma_w / \bar{g}\end{aligned}\tag{4-69}$$

where

G_s = specific gravity of solids

S = saturation

e = void ratio .

Substituting these into equation 4-66 yields

$$\frac{\partial(se)}{\partial t} = \frac{(1+e)}{\gamma_w} \frac{\partial}{\partial x} \left(\frac{\bar{k} S}{1+e} \frac{\partial u_f}{\partial x} \right) + \frac{(1+e)\bar{g}}{\gamma_w} \gamma_f \quad (4-70)$$

If the sample is totally saturated then equation 4-70 is the same as the one developed by Gibson et al (1967). If, in addition, constant permeability and small deflections are assumed, equation 4-70 becomes the classical Terzaghi equation. Expanding and rearranging the terms of equation 4-70 yields:

$$\begin{aligned} \frac{\partial e}{\partial t} = & \frac{(1+e_0)^2}{(1+e)\gamma_w} \left(\bar{k} \frac{\partial^2 u_f}{\partial x^2} + \left(\frac{\bar{k}}{s} \frac{\partial s}{\partial x} + \frac{\partial \bar{k}}{\partial x} - \frac{\bar{k}}{(1+e)} \frac{\partial e}{\partial x} \right) \frac{\partial u_f}{\partial x} \right) \\ & + \frac{(1+e)\bar{g}}{s\gamma_w} \gamma_f - \frac{e}{s} \frac{\partial s}{\partial t} \end{aligned} \quad (4-71)$$

The number of variables in the equation must be reduced before it can be solved. The degree of consolidation is defined as

$$U = \frac{e_0 - e}{e_0 - e_1} \quad (4-72)$$

and a function λ is defined as

$$\lambda = 1 - \sigma = \frac{e - e_1}{e_0 - e_1} = \frac{e'}{\bar{a}_v \Delta p} \quad (4-73)$$

where

$$\begin{aligned} \bar{a}_v &= \text{average compressibility} \\ e &= \text{void ratio} \\ e_0, e_1 &= \text{initial and final void ratio} \\ e' &= e - e_1 \\ \Delta p &= \text{load increase.} \end{aligned}$$

Now the time derivative of λ is

$$\frac{\partial \lambda}{\partial t} = \frac{1}{e_0 - e_1} \frac{\partial e}{\partial t} = \frac{1}{\bar{a}_v \Delta p} \frac{\partial e'}{\partial t} \quad (4-74)$$

In order to have a dimensionless fluid pressure and distance, the following functions are defined:

$$\begin{aligned} u &= u_f / u_{f_0} \\ Z &= X/H \end{aligned} \quad (4-75)$$

where u_{f_0} is the initial pore pressure and H is the original sample height. Substituting equations 4-73 and 4-74 into 4-71 yields

$$\begin{aligned} \frac{\partial \lambda}{\partial t} &= \frac{(1+e_0)^2}{(1+e)} \frac{u_0/\Delta p}{\gamma_w \bar{a}_v H^2} \left(\bar{k} \frac{\partial^2 u}{\partial Z^2} + \left| \frac{\bar{k}}{s} \frac{\partial s}{\partial Z} + \frac{\partial \bar{k}}{\partial Z} \right. \right. \\ &\quad \left. \left. - \frac{\bar{k}}{1+e} \frac{\partial e}{\partial Z} \right| \frac{\partial u}{\partial Z} \right) + \frac{1+e}{s \gamma_w} \psi^f - \frac{e}{s} \frac{\partial s}{\partial t} \end{aligned} \quad (4-76)$$

This general fluid flow equation for a two phase fluid along with the creep equation will describe the behavior of the material.

The creep function will be expressed by equation 4-66 which is

$$\frac{d\epsilon}{dt} = \phi \sinh(\phi_c \tau) \quad (4-77)$$

The stress equation for a material with a large amount of creep or structural viscosity is given as

$$\Delta p - \bar{\sigma} = u_f + \tau \quad (4-78)$$

where

Δp = load increase

$\bar{\sigma}$ = effective stress

u_f = fluid pressure

τ = viscous resistance

Equation 4-77 can be rearranged to give

$$\tau = \frac{1}{\phi_c} \sinh^{-1} \left(\frac{\dot{\epsilon}}{\phi} \right). \quad (4-79)$$

Substituting equations 4-70, 4-75 and 4-78 into 4-77 yields

$$\lambda = \frac{1}{\phi_c \Delta p} \sinh^{-1} \left(\frac{\dot{\epsilon}}{\phi} \right) + \frac{u_f}{\Delta p} \quad (4-80)$$

which can be rearranged to give

$$\dot{\epsilon} = \phi \sinh\left[\left(\lambda - \frac{u_f}{\Delta p}\right) \phi_c \Delta p\right] \quad (4-81)$$

Substituting of 4-74 gives

$$\frac{\partial \lambda}{\partial t} = \frac{\phi}{\bar{\alpha}_v \Delta p} \sinh\left[\left(\lambda - \frac{u_f}{\Delta p}\right) \phi_c \Delta p\right] \quad (4-82)$$

A creep time factor

$$T_s = \frac{\phi \phi_c}{\bar{\alpha}_v} t \quad (4-83)$$

can be introduced into equation 4-82 to give

$$\frac{\partial \lambda}{\partial T_s} = \frac{1}{\Delta p \phi_c} \sinh\left[\left(\lambda - \frac{u_f}{\Delta p}\right) \phi_c \Delta p\right] \quad (4-84)$$

The two equations, 4-76 and 4-84, describe the general behavior of the material. They must be evaluated simultaneously because both processes occur from the initiation of load increase. The method used to solve these is the same as that developed by Barden (1965a, 1968, 1969) and is outlined in Appendix A. The solution involves the use of finite difference equations. Chapter V will discuss the different variables involved in the solution of these equations.

CHAPTER V

MATERIAL PROPERTIES

The two equations needed to model the consolidation behavior of milled refuse have been developed in the previous chapter in terms of several variables. The interdependence of some of these variables will be discussed in this chapter so that the functions can be expressed in more convenient terms.

Fluid Transmissibility

The laminar flow of a fluid through a porous medium is described by Darcy's law, which says that the velocity and hydraulic gradient of the fluid are related by a constant of proportionality. Except for fine clay and coarse gravel, this relationship governs flow in soils and is usually assumed to be valid for peats. The constant of proportionality depends primarily on the degree of saturation and the size and geometry of the pore spaces. Generally, this constant is called the coefficient of permeability, as it will be herein for the totally saturated case. The general coefficient for both saturated and unsaturated flow will be called the transmissibility, which is a function of permeability and saturation. To avoid confusion, it should be noted that the term "coefficient of transmissibility" is sometimes used in hydrology to mean the rate of flow of water through a vertical section of an aquifer 1 ft wide, and extending over its total thickness, under a hydraulic gradient of unity at 60° F (Wisler and Brater, 1959).

Possible relationships between the void ratio and the permeability, and some suggested functional relationships for evaluation of transmissibility will be developed in the following paragraphs.

Permeability - Pore Size

The classical approaches to the variation of permeability with geometry are based upon one simplifying assumption: that the flow of a fluid through randomly sized and spaced voids which are randomly connected to other voids can be characterized by the flow through an equivalent system of single sized voids. The two most used theories for making these approximations are those of Poiseuille and Kozeny (Millington and Quirk, 1961). Poiseuille developed a relationship involving average fluid velocity, gradient and radius for laminar flow in a capillary tube and found that

$$V_{av} = \frac{\gamma_w \pi A R_f^4}{8 \pi} i = \frac{\gamma_w R_f^2}{8 \pi} i \quad (5-1)$$

where

V_{av} = average velocity

γ_w = unit weight of water

R_f = tube radius

i = hydraulic gradient

A = area

π = coefficient of dynamic viscosity.

This approach is applied to a porous material by using a bundle of different sized capillary tubes as a model to represent flow in such a medium.

Kozeny's equation describes a porous medium in terms of particle size distribution rather than the pore size distribution. This approach assumes that permeability can be related to the packing and size of the components of the matrix and his equation is

$$k = c \frac{d n^3}{(1 - n)^2} \quad (5-2)$$

where

k = permeability

d = measure of particle size

n = porosity

c = empirical constant

Other expressions based on the grain size distribution have been developed by different persons, but they are nearly the same as those presented and have the same limitations.

Both the tube and grain size approaches yield fair results for a given material under given conditions, but do not lend themselves to any broad generalizations. Neither method takes the natural structuring of a material into account, such as bedding in sand and silt or fissures in clay.

Childs (1948), Childs and Collis-George (1950) and Collis-George (1953) used statistical methods and Poiseuille's capillary tube model to develop analyses for predicting permeability and the effect of saturation on transmissibility. In their analysis, the statistical distribution of pore sizes of a material is determined from the results of a water content versus suction analysis. With this information the contribution of each of the pores is determined and the transmissibility of the material is found by summation of the effects of each of the pores. Using this approach, better approximations to permeability are obtained for a wider class of materials than before. Two modifications of the Childs - Collins-George model are

proposed by Marshall (1958) and Millington and Quirk (1959, 1961). Both used an equation of the form

$$k = 1.884 \times 10^4 n^c j \left[h_1^{-2} + 3h_2^{-2} + \dots + (2j - 1) h_j^{-2} \right] \quad (5-3)$$

where

1.884×10^4 = composite of constants to give hydraulic conductivity in terms of cm-min^{-1} and to convert effective pore radius to pressure potential in m.b. at 27°C

n = porosity

h_i = potential of a given pore radius class

c = constant

j = number of classes of pores

Marshall uses a constant c , equal to two, and j is the number of pore classes up to the size of interest, while Millington and Quirk use $c = 4/3$ and j represents the total number of pore intervals. Except for these two constants, the two methods are identical.

Jackson, Reginnato and Van Bavel (1965) compared Childs' and Collis-George's model and the two modifications with experimental results, and found that the Millington and Quirk method gave the general shape of the experimental curve although it overestimated the values. Using a matching factor, this method yielded results that agreed very well with actual test results. However, the other methods, though they did not yield the best fitting shape of the curve, did predict some of the actual permeability values.

The bulk of the work presented thus far is the product of agronomists who are primarily interested in the water-energy relationship aspects of permeability and void ratio. Soil mechanicians need to know the variations

in permeability with void ratio as consolidation, or changes in the void ratio occur. Consolidation produces a rearrangement of the solid matrix of a material, which reduces the cross-sectional area of the flow channel causing a reduction in permeability. Classical consolidation theory assumes that the average permeability exists throughout the test and the net change is rather small. However, with great change in permeability, as reported by Hanrahan (1954), and the direct permeability versus void ratio dependence indicated by the proposed models, the changes in permeability cannot be neglected when the consolidation produces finite strains.

Based on the Poiseuille model, several workers in the field of soil mechanics have proposed different permeability-void ratio relationships. Taylor (1948) gives one relationship as

$$k_1 : k_2 = \frac{c_1 e_1^3}{1 + e_1} : \frac{c_2 e_2^3}{1 + e_2} \quad (5-4)$$

where

k_1, k_2 = permeabilities at different void ratios

e_1, e_2 = different void ratios

c_1, c_2 = shape factors

For some soils the shape factors are nearly one and the permeability is only a function of void ratio. Several other relationships of the form

$$k = k_1 F(e) \quad (5-5)$$

have been proposed. Some expressions for $F(e)$ are

$$F(e) = \frac{2e^2}{1+e}$$

$$F(e) = \frac{2e^3}{1+e} \quad (5-6)$$

Terzaghi and Peck (1967) report that A. Casagrande proposed in an unpublished paper that in a clean sand with bulky grains the permeability is related to the void ratio by

$$k = 1.4 e^2 k_{0.85} \quad (5-7)$$

where $k_{0.85}$ is permeability at a void ratio of 0.85. Means and Parcher (1963) give an equation developed by Terzaghi as

$$k = k_o C_1 (e - e_o)^2 \quad (5-8)$$

where C_1 is a shape factor and e_o , k_o are the void ratio and permeability at a given density. A graph of these different relationships is shown in Figure 5-1. Mitchell et al (1965) arrive at the same relationship as the cubic function in equation 5-6.

The lack of use of the different void ratio-permeability relationships in the existing studies of consolidation theory is due to several factors. First, inclusion of these functions will introduce considerable mathematical complications. Also, there is a general lack of accuracy in the determination of the coefficient of consolidation that will partially negate the advantage gained in use of these relationships. Last, for small deflections in materials with low void ratios the variation in the compressibility tends to counterbalance the influence of the permeability changes. Schiffman

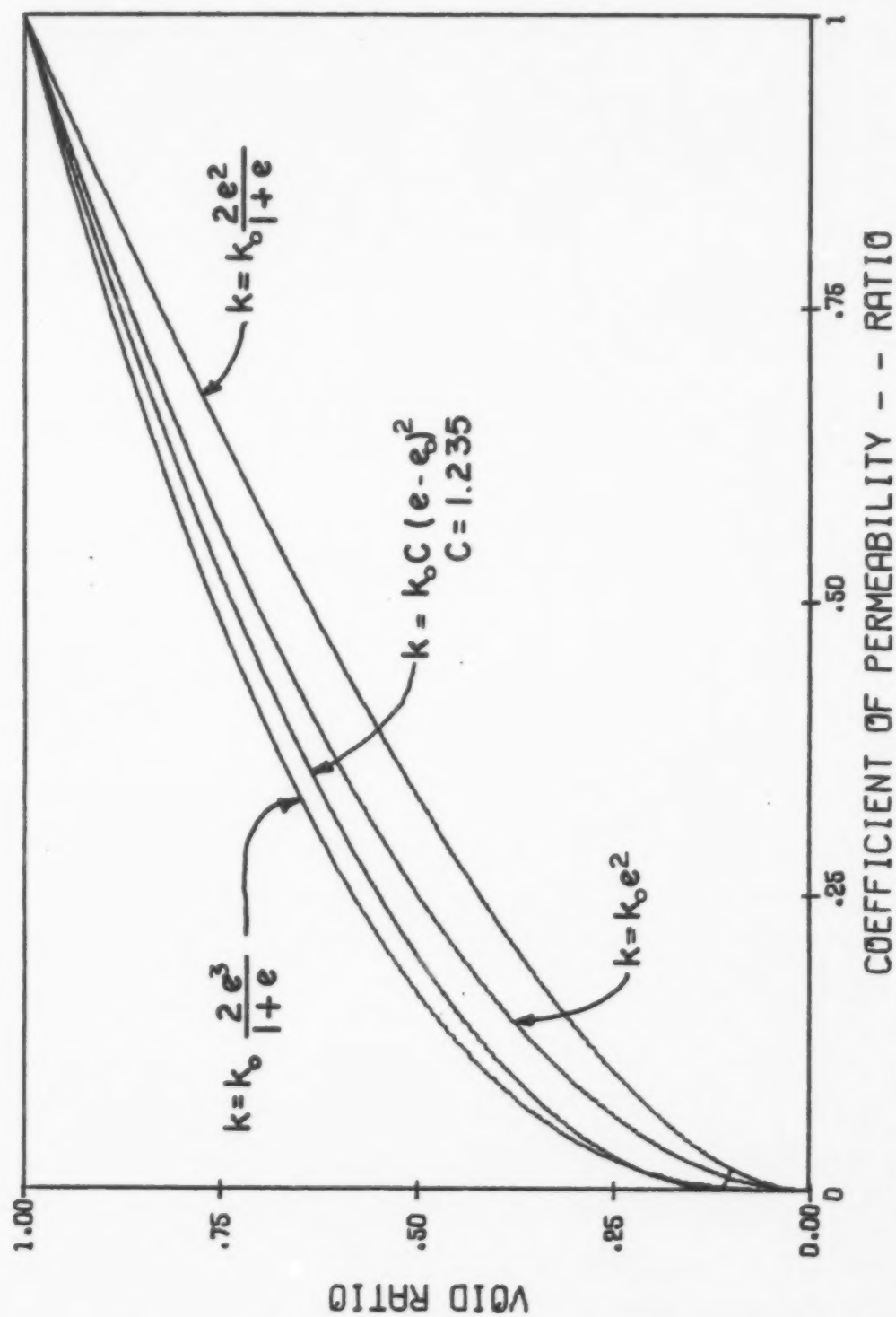


Figure 5-1: Variation in permeability void ratio relations for void ratio less than unity (after Means and Archer).

(1958), Barden (1965a), Barden and Berry (1965) relate void ratio change to pore pressure and use the linear function

$$k = a u + k_f \quad (5-9)$$

Raymond (1966, 1969) and Poskitt (1969, 1971) develop permeability-effective stress relations based on an empirical relation between void ratio and the logarithm of permeability, of the form

$$k = k_f \left(1 - \frac{u}{\sigma_T}\right)^{-C} \quad (5-10)$$

where

- k_f = final permeability
- u = excess pore pressure
- σ_T = total stress
- C = constant .

Examination of the different proposed relationships between void ratio and permeability suggests that they differ by a great deal. The more general equations (5-6 to 5-9) are developed for the void ratio range of zero to unity, while those for use in consolidation analysis are for a more limited void ratio range. Any of the curves shown in Figure 5-1 can be accurately approximated by a linear function over a small void ratio increment, and there is little difference between any of them over the entire range. If the equations in Figure 5-1 are plotted for large void ratios, the close agreement between the functions is lost as shown in Figure 5-2. Lea and Brawner (1963) and Hanrahan (1954) give permeability versus void ratio for

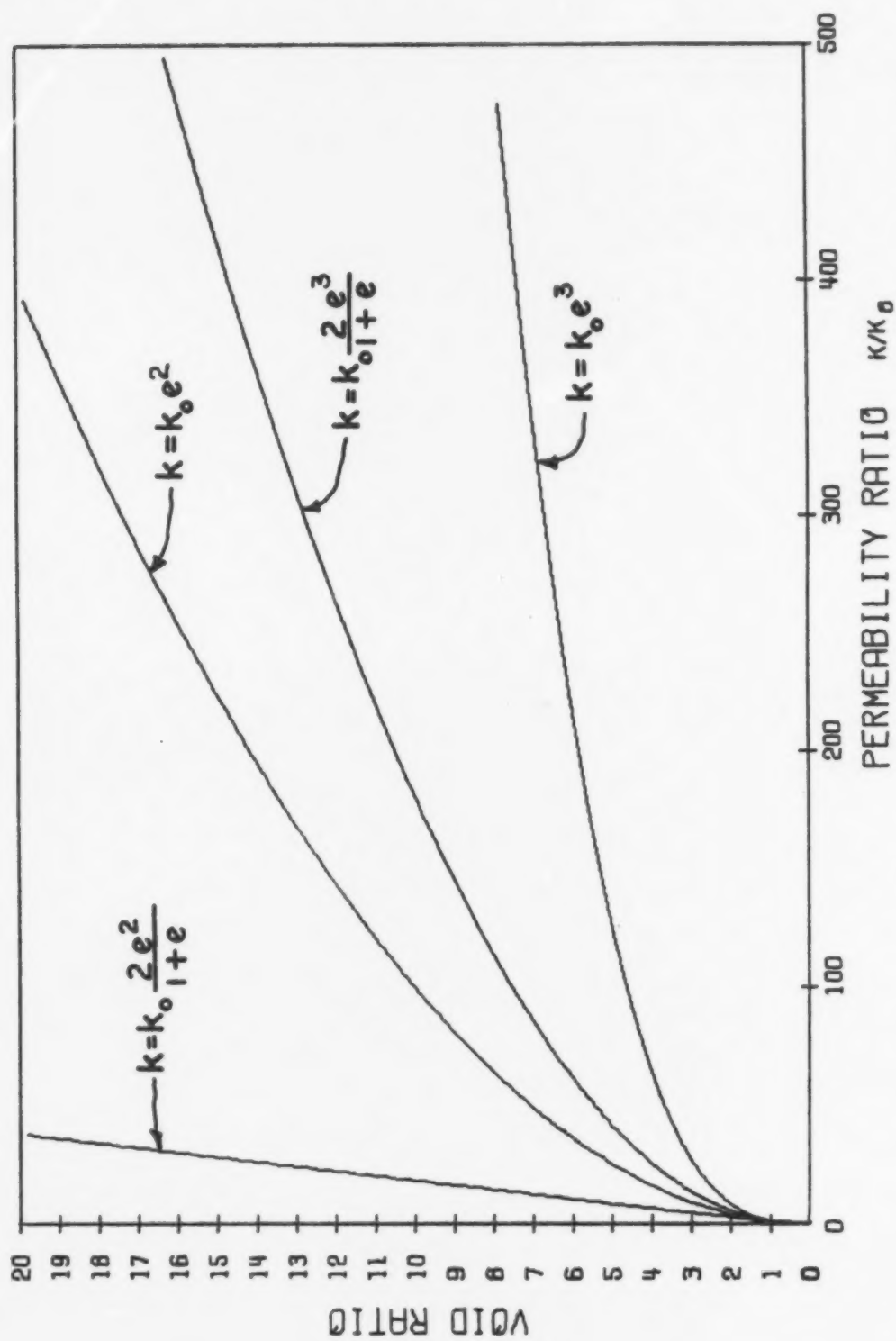


Figure 5-2: Variation in permeability - void ratio relations for void ratios greater than unity.

peat with large void ratios. Figure 5-3 shows Lea and Brawner's results for many tests, and indicates that there is a linear relationship between the logarithm of void ratio and the logarithm of permeability. Hanrahan's data, when plotted in the same way, show a nearly linear relationship for most of the graph, Figure 5-4, but there are very few tests.

The results of these limited data would indicate a nearly linear log void ratio versus log permeability relationship. When the functions from Figure 5-1 and 5-2 are replotted on a log void ratio versus log permeability scale, they all yield a straight line, indicating that a power of void ratio function may provide a satisfactory relationship between the two variables. In the parallel capillary tube model of Poiseuille, permeability is a function of the fourth power of the radius of the pores and, since for a given material the void ratio and pore radius are likely to be directly related, permeability should be approximately a function of a power of the void ratio. Figure 5-5 shows the data from these studies, along with void ratio functions of the form

$$k = k_o \left(\frac{1 + e_o}{e_o^{N'}} \right) \left(\frac{e^{N'}}{1 + e} \right) \quad N' = 5, 6, 7 \quad (5-11)$$

plotted as void ratio versus permeability ratio k/k_o . The data and functions are plotted using $e_o = 12$. The graph shows that the permeability function for $N' = 5$ follows one of Lea and Brawner's curves quite well, while $N' = 6$ seems to give the best fit for the available data.

Transmissibility-Saturation

A given material with a given void structure, when saturated, will pass

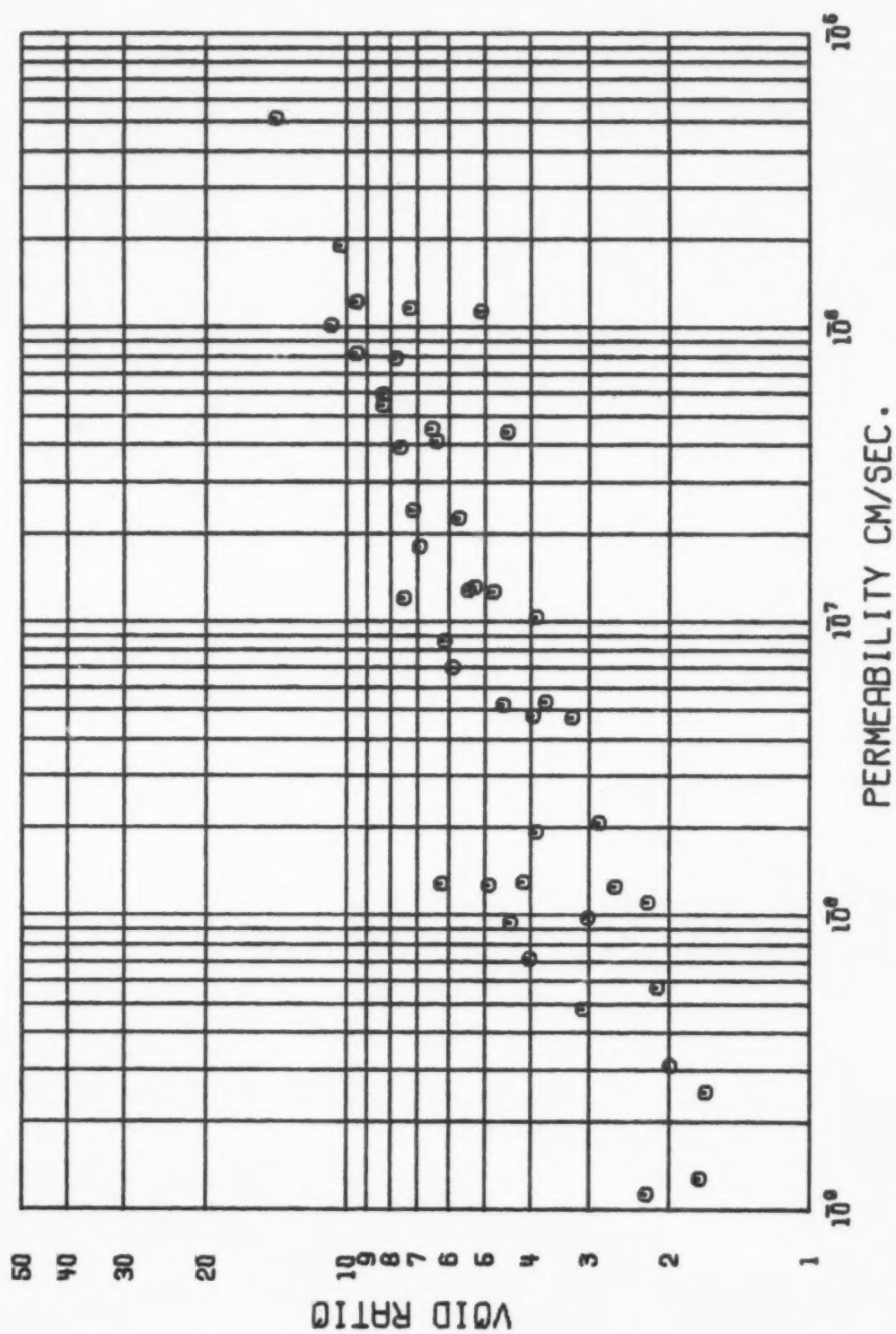


Figure 5-3: Lea and Brawner's permeability test results.

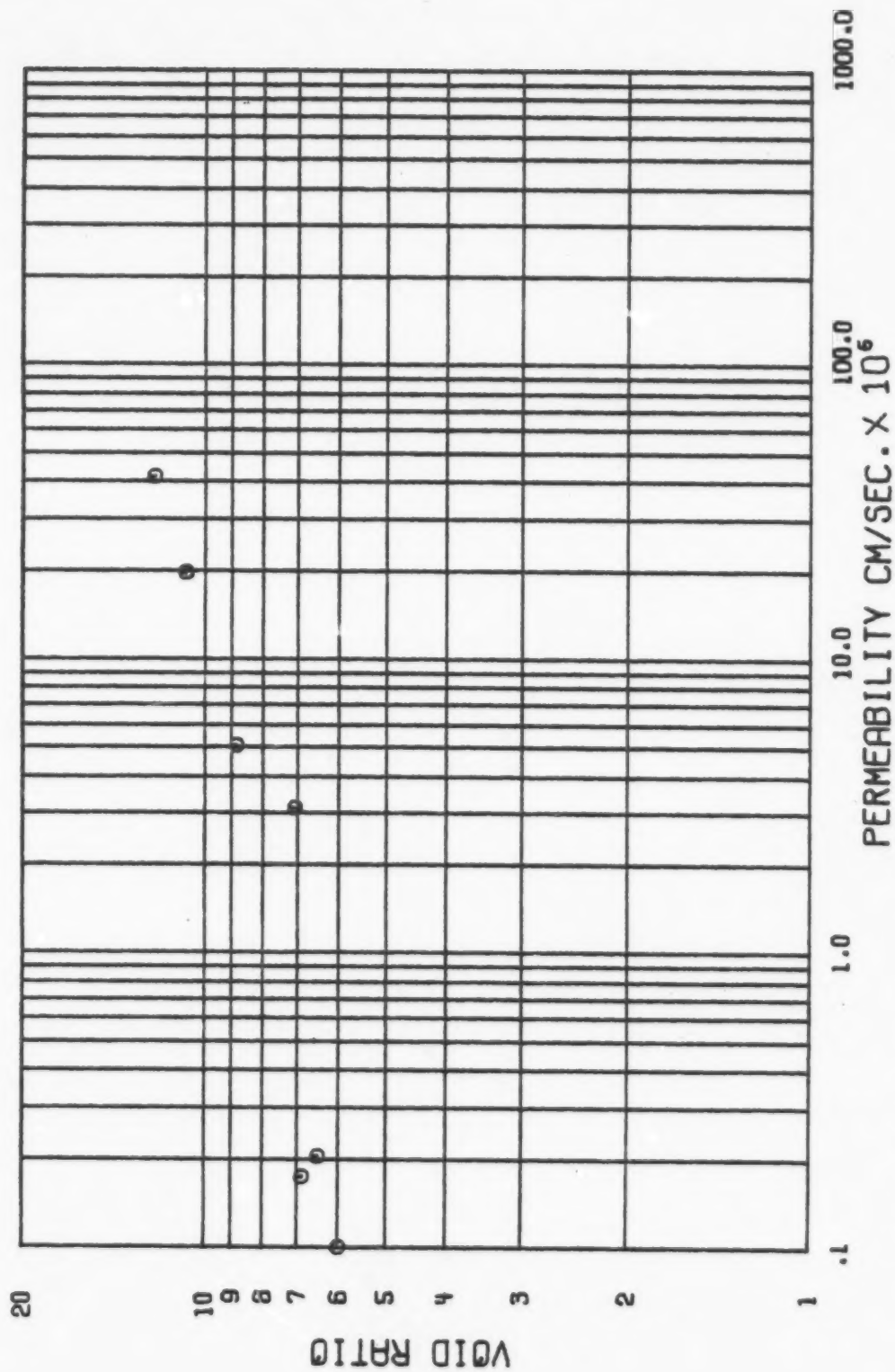


Figure 5-4: Hanrahan's permeability results.

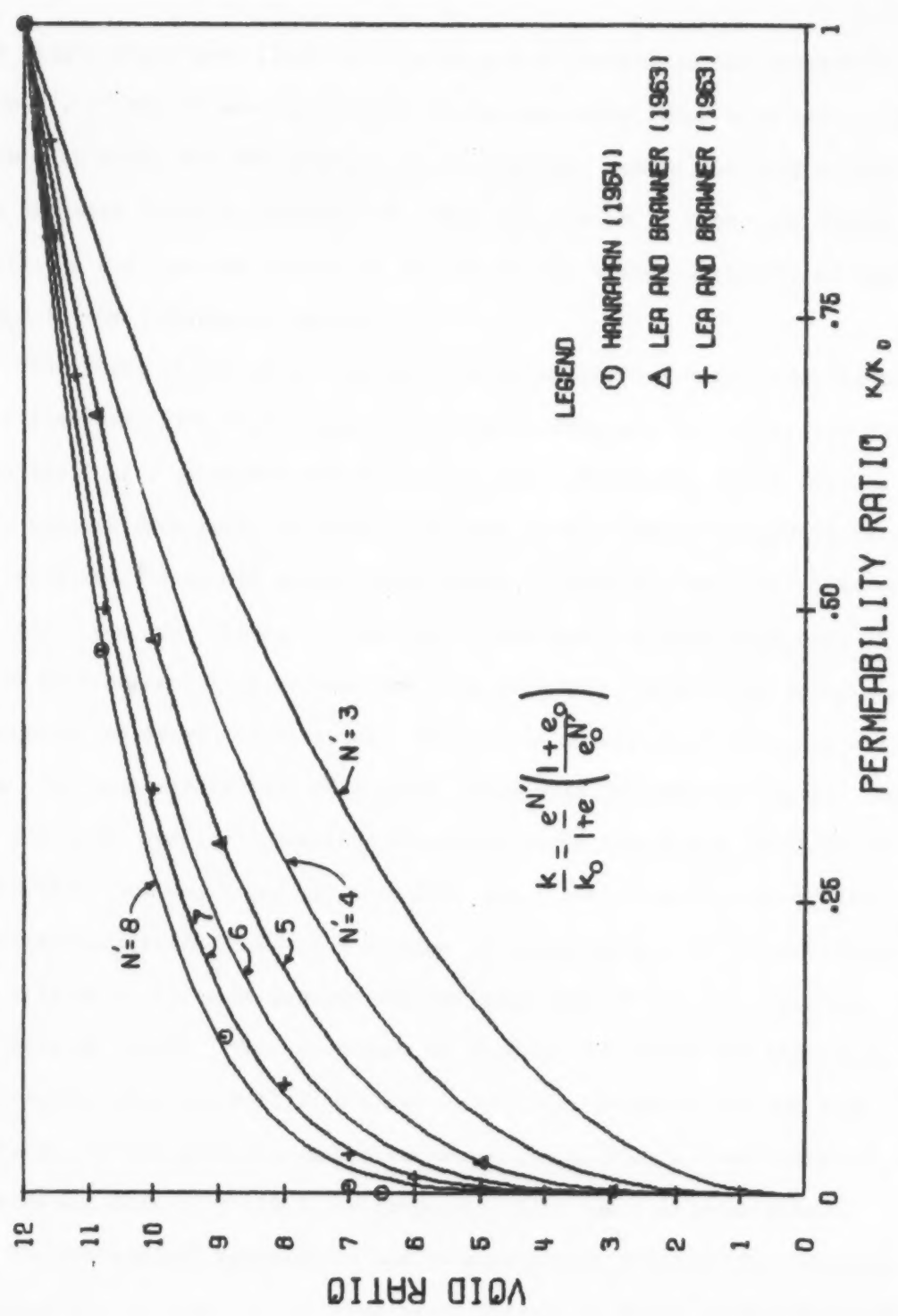


Figure 5-5: Actual and theoretical permeability - void ratio relations.

the single phase pore fluid at a certain rate defined as the permeability. However, if one or more additional fluids are added, they will interfere with each other and the original fluid, and will reduce the rate at which the original fluid is transmitted. For the problem at hand, the fluids are water and air, and the effect of the air on the transmissibility of the water is the information needed.

The effect of air or gas on the flow of a liquid through a porous medium is disproportionate to the amounts present. When gas and water form the pore fluid of a permeable medium, the actual composition of the mixture will vary greatly from point to point. Because of the effect of surface tension, gas will collect in the larger pores first, disrupting the best fluid conductors. The distribution of the gas is unstable, as with flow some will break from concentrated volumes and flow with the liquid to be deposited elsewhere, or leave the material. Thus in an unsaturated, or multi-phase flow, the transmissibility of a given fluid will continually change. For the air-water mixture there is a threshold water saturation above which water will flow and below which it will not. The value of this differs for different materials. Clay has a value of approximately 85 percent, while for silt it is 40 to 50 percent and for sand near 20 percent (Jennings and Burland, 1963). The importance of this is that below the threshold saturation, only gas will flow under excess pore pressure and the time required for the pore pressure dissipation is negligible. The work of Yoshimi and Osterberg (1963) on compacted silty clay verifies this.

The theoretical approach to the transmissibility-saturation behavior of materials is based on the same basic Poiseuille model as was the permeability analysis. Childs (1953) assumes that at a certain saturation only

pores of a specific radius and smaller can contain water. With knowledge of the size distribution of pores, the transmissibility can be calculated for the remaining flow area. Marshall (1958) and Millington and Quirk (1959, 1961) used the same model as Childs and found that they could also predict the transmissibility-saturation relationship for materials. Brutsaert (1963, 1966, 1968) says that greater insight to the transmissibility problem can be obtained if certain probability relationships are used to describe the pore size distribution and that their use will produce a closed form solution. To allow for the residual saturation, or the water that will not flow from the material, he introduces into his analysis the linear transformation

$$S_e = \frac{S - S_r}{1 - S_r} \quad (5-12)$$

where

S_e = effective saturation

S = degree of saturation

S_r = the residual saturation, or the saturation at which the transmissibility becomes zero

Brutsaert calculates the transmissibility of different materials using several different pore size distributions and found that they compare quite well with the values obtained by Jackson et al (1965). Brutsaert (1968) points out that the transmissibility calculated from "series-parallel model is relatively insensitive to the particular probability law used to describe the particular distribution of the pores."

Figure 5-6 shows the results of Brutsaert's calculations based on his

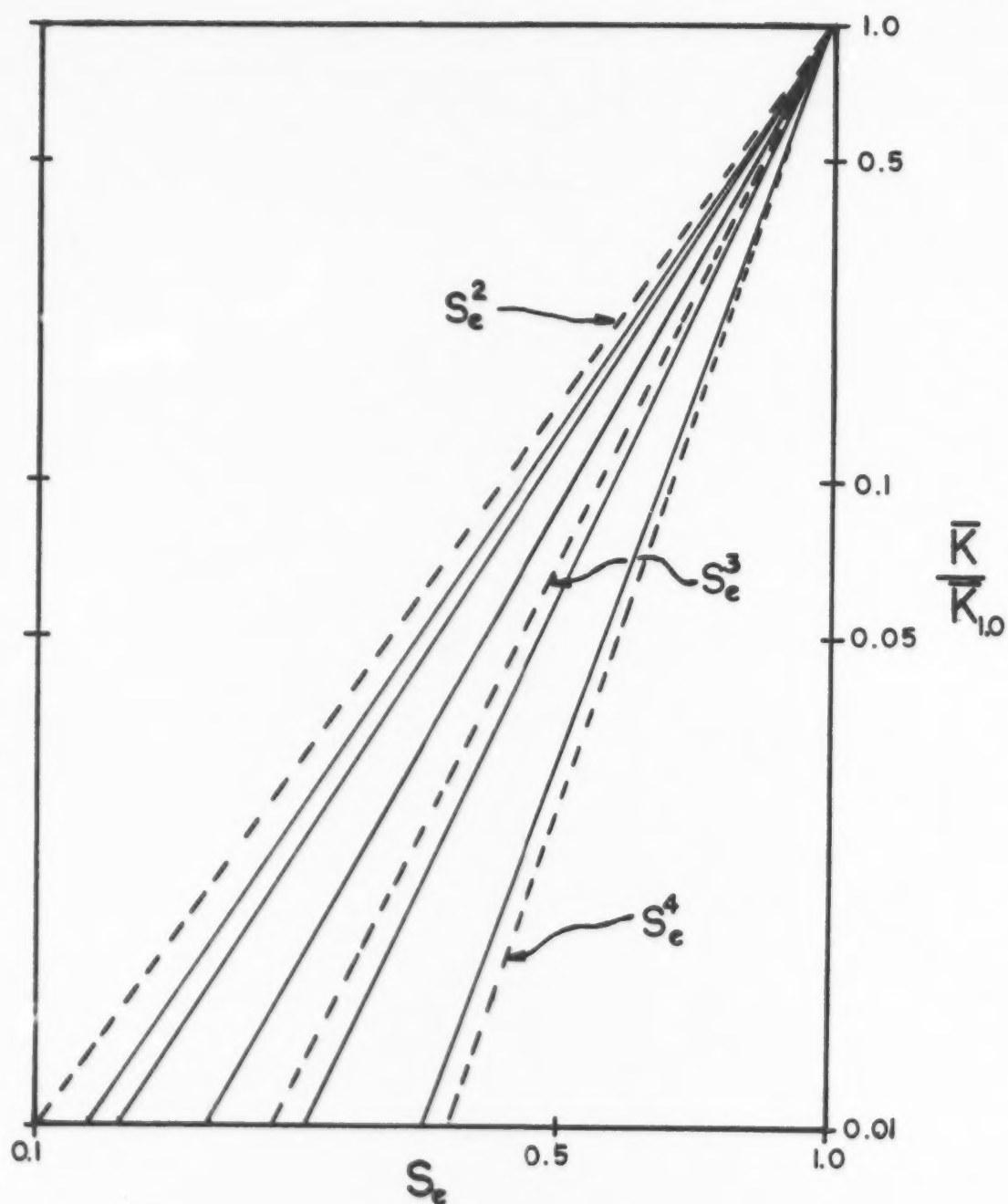


Figure 5-6: Brutsaert's transmissibility-saturation functions.

proposed model as well as a simple power function of the relative saturation. The solid lines represent the transmissibility-relative saturation function based on the theoretical model presented in the last of his three articles, while the dashed lines are those for the function

$$\bar{K} = k S_e^{N_s} \quad (5-13)$$

where

$\bar{K}$ = transmissibility

k = permeability

N_s = constant

S_e = effective saturation .

As can be seen on the graph, the proper value of the power for the effective saturation will yield the same curve as the complex relationships developed from the parallel capillary tube model. Brutsaert says that for "a medium with very uniform pore sizes, N_s may be as small as 2, whereas for a medium with a wide distribution of the pore sizes it may exceed 5." The exact determination of this constant is not of utmost importance because, as shown by Ibrahim and Brutsaert (1968), the rate of infiltration into a loam changes very little for N_s equal to 3, 3.5 or 4.

As with the geometry-permeability relationships, most of the analysis of unsaturated flow has been conducted by agronomists, while soil mechanicians have given little attention to the subject. Yoshimi and Osterberg (1963) studied compacted silty clay in which the saturation was less than the residual saturation value and thus involved only the flow of pore air. Mitchell, Hooper and Campanella (1965) developed an equation for the

transmissibility using the capillary tube model and assuming that the gas is uniformly distributed across the flow area. The resulting equation is

$$\bar{k} = \frac{\gamma_w}{\eta} \frac{C_s V_s^2}{A_s^2} \frac{e^3}{1+e} S^3 \quad (5-14)$$

where

$\bar{k}$ = transmissibility

C_s = shape constant

V_s = volume of solids

A_s = cross-sectional area of pores that carry fluid

e = void ratio

S = degree of saturation

η = viscosity of water

γ_w = unit weight of water

Barden (1965a), although he dealt with unsaturated flow, considered the saturation to be constant for each problem and thus it has no effect on the transmissibility in a given case. Toriyama and Sawada (1968) assumed that water flow is related to saturation in a relationship similar to equation 5-14. Their relationship is

$$\bar{k} = k \frac{S_r^{N_j} - S_r^{N_j}}{1 - S_r^{N_j}} \quad (5-15)$$

where N_j is a constant.

Examination of equations 5-13, 5-14, and 5-15 shows a great deal of similarity. Both 5-13 and 5-14 have been experimentally verified and, in

fact, 5-14 is a special case of 5-13. Expansion of equation 5-15 for $N_j \geq 2$ yields several more terms than equation 5-13. Either of these two equations should provide a general equation which will relate the transmissibility to the saturation. The general transmissibility function depends on several variables, the two most important being void ratio and saturation. By use of equations 5-11 and 5-13, the transmissibility becomes

$$\bar{K} = k_o \frac{(1 + e_o)}{e_o^{N'_1}} \frac{e^{N'_1}}{(1 + e)} S_e^{N_s} \quad (5-16)$$

If $S = 100$ percent, then

$$\bar{K} = k_o \frac{e^{N'_1}}{1 + e} \frac{1 + e_o}{e_o^{N'_1}} \quad (5-17)$$

Compressibility-Void Ratio

Compressibility of a material decreases as it is compacted, or, in other words, its resistance to further compaction increases. Classical consolidation theory assumes that a material's compressibility is constant, which is nearly valid for the small deflections considered. However, when an analysis of a system with finite movements is made, the changes in the compressibility must be taken into account. The compressibility of peat may decrease by a factor of 20 during consolidation (MacFarlane, 1969). Thus, the effects of the increase in deformation resistance during finite deformation will be of great importance in the study of these movements.

Several studies of classical type consolidation deal with the variation of the compressibility of a clay. Davis and Raymond (1965) assume that a

linear inverse relationship between the effective stress and the compressibility exists. Poskitt (1969) makes the same assumption. The void ratio range studied in these investigations is rather limited; therefore these same assumptions cannot be carried over to a large displacement study. A few studies report compressibility changes over a wide range of void ratios for different peats and some organic soils. Lea (1958), Lea and Brawner (1963) and Tessier (1966) give results of consolidation tests showing the compressibility-void ratio relationship for void ratios as great as 14. Moran et al (1958) gives the same information for some organic soils over about half this void ratio range. Figure 5-7 shows the results of those studies, while 5-8 shows the same data for a lesser range of values. Figure 5-9 shows the data from 5-7, along with several possible curves that might establish a relationship for compressibility as a function of void ratio. A very good fit is obtained from the function

$$a_v = e^3 / e_{10}^3 a_{v10} \quad (5-18)$$

where

a_v = compressibility

e = void ratio

$a_{v10} = a_v$ for void ratio = 10 .

However, the curves for

$$a_v = \frac{e^{N''}}{e_{10}^{N''}} a_{v10} \quad N'' = 2, 4 \quad (5-19)$$

are also good approximations.

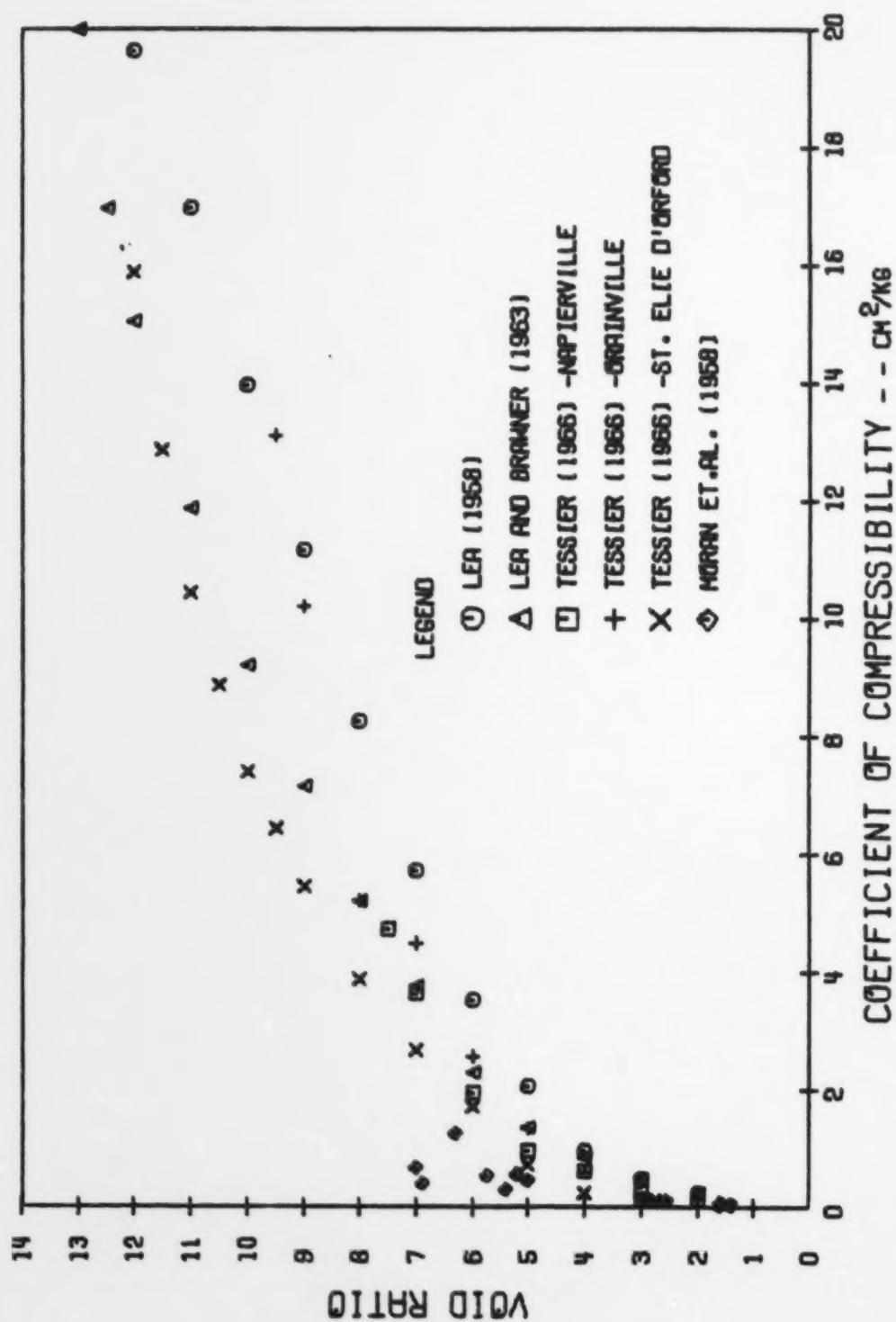


Figure 5-7: Variation in compressibility with void ratio.

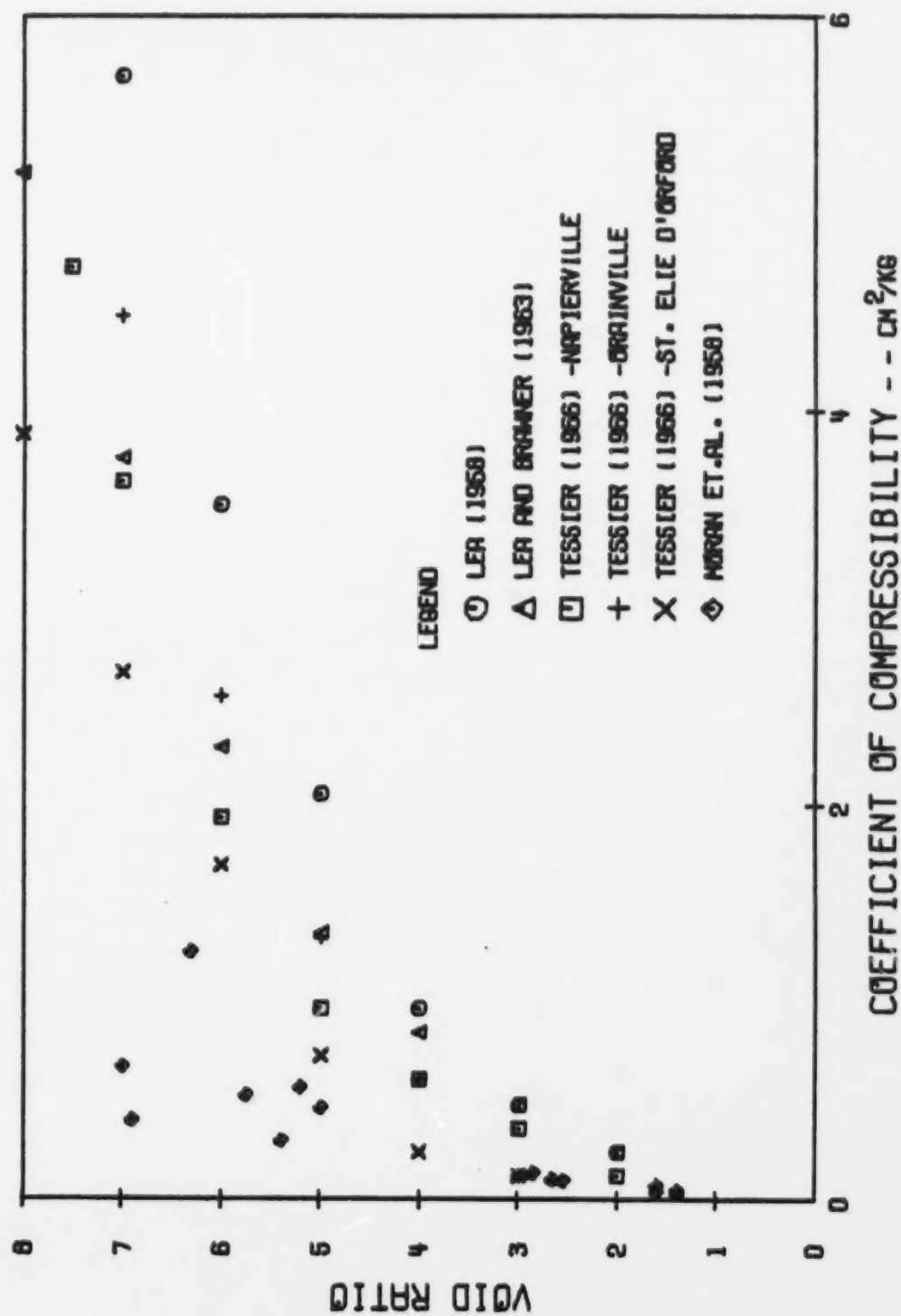


Figure 5-8: Variation in compressibility with void ratio - for limited void ratio range.

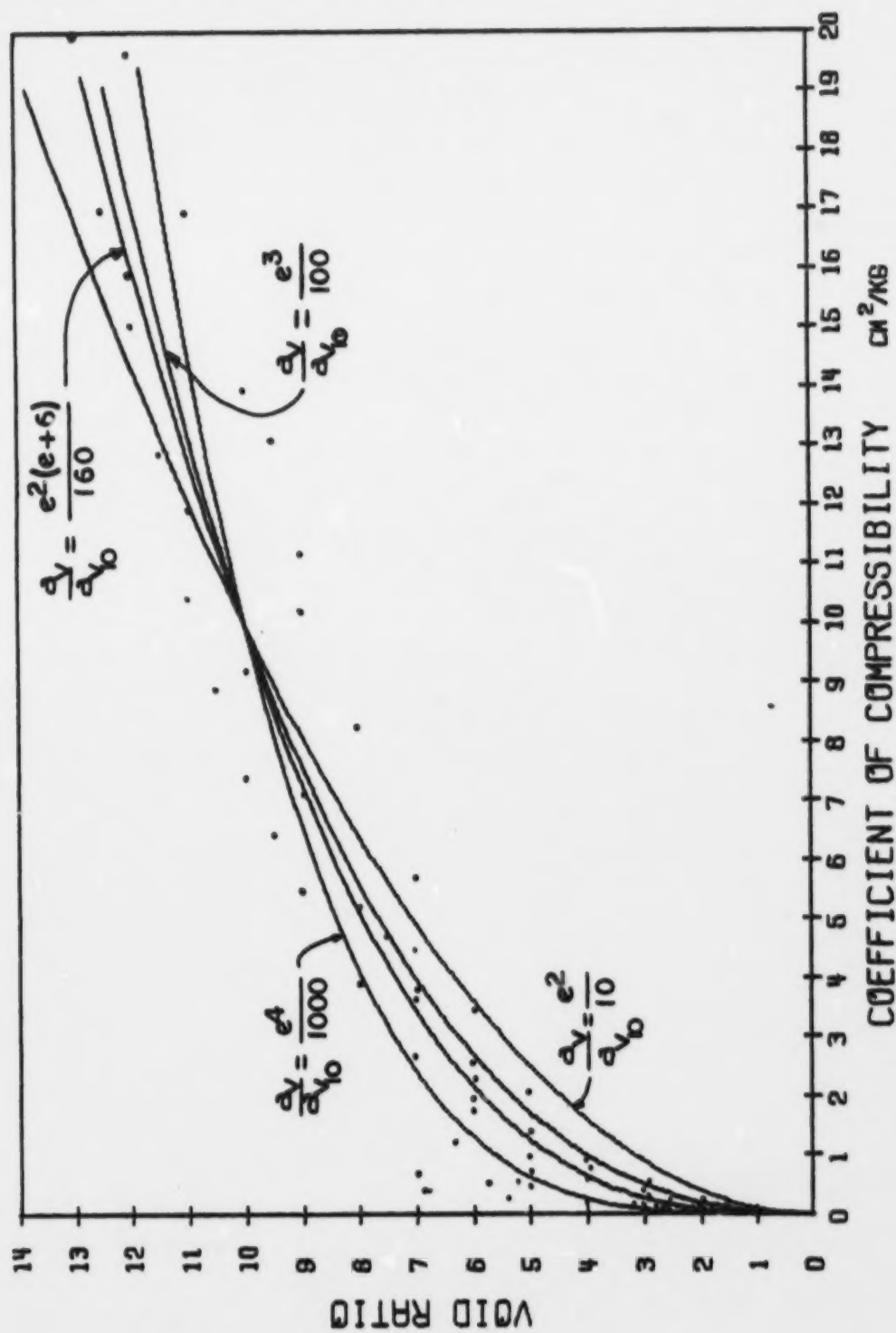


Figure 5-9: Possible relationships for compressibility - void ratio function.

The data shown in Figures 5-7 and 5-8 are plotted in Figures 5-10 to 5-15 as the ratio of the coefficient of compressibility corresponding to a given void ratio to the maximum value of the coefficient for the data set. The ratio is

$$F = \frac{a_v}{a_{v \max}} \quad (5-20)$$

Also shown on these figures are the functions

$$\frac{a_v}{a_{v \max}} = \frac{e^{N''}}{e_{\max}^{N''}} \quad (5-21)$$

for N'' values 2, 3 and 4. Figure 5-10 gives a very good fit for $N'' = 3$ in equation 5-23, while the data on 5-11 in general lies somewhere between $N'' = 2$ and $N'' = 3$. The data of Moran et al do not follow any of the curves as well as the data of other workers do. As can be seen in Figure 5-12, their results do follow the general trend. Figures 5-13, 5-14, and 5-15, Tessier's data, all seem to fit between $N'' = 3$ and $N'' = 4$. In general, equation 5-21 will fit the data quite well for a given N'' value.

As consolidation progresses, the compressibility will decrease, but the differential void ratio change will still be equal to the instantaneous compressibility times the change in effective stress or

$$de = a_v d\bar{\sigma} \quad (5-22)$$

If the instantaneous compressibility is defined by equation 5-19, then equa-

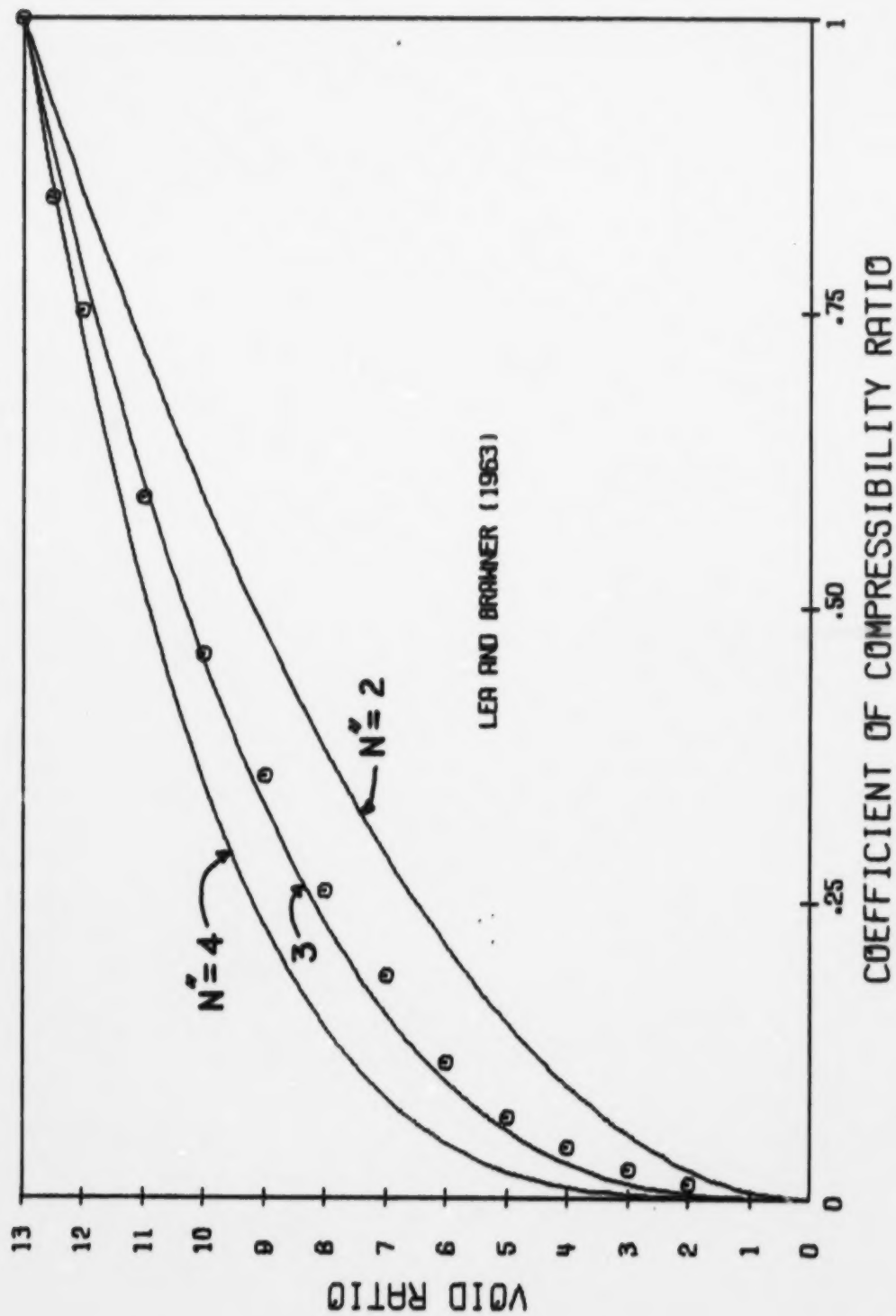


Figure 5-10: Fit of Lea and Brauner's (1963) data to proposed compressibility function.

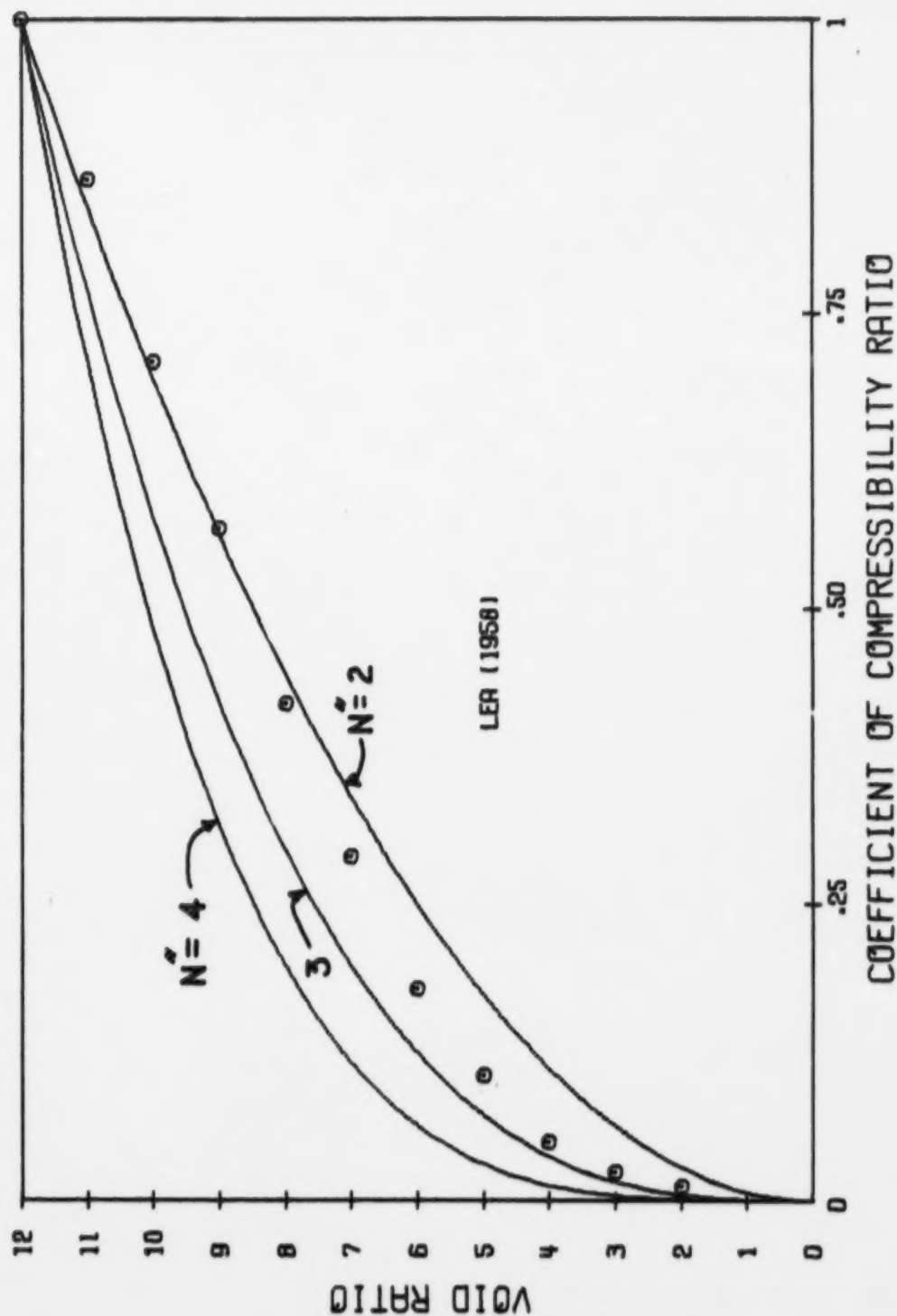
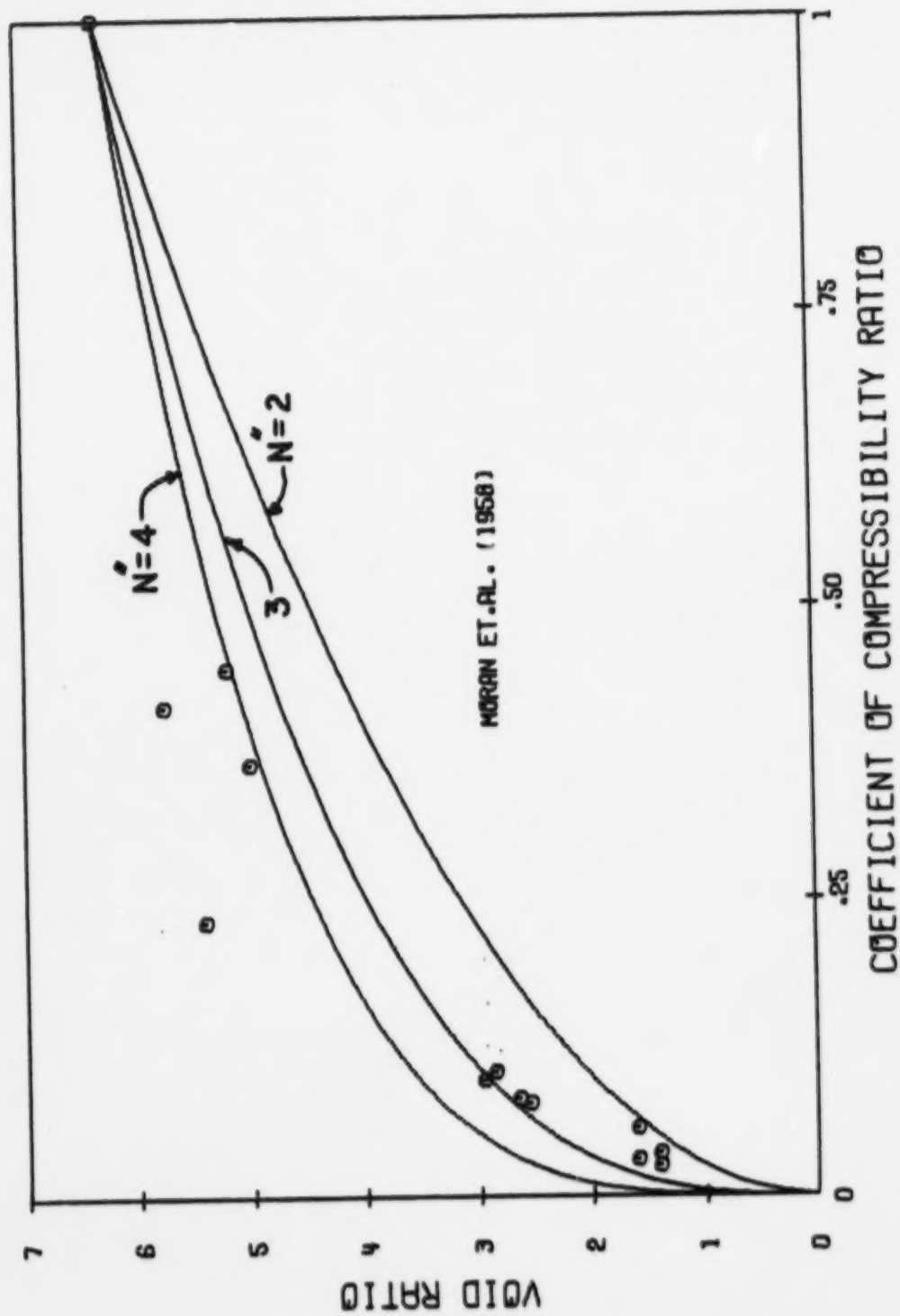


Figure 5-11: Fit of Lea's (1958) data to proposed compressibility function.



MORAN ET AL. (1958)

Figure 5-12: Fit of Moran et.al. (1958) data to proposed compressibility function.

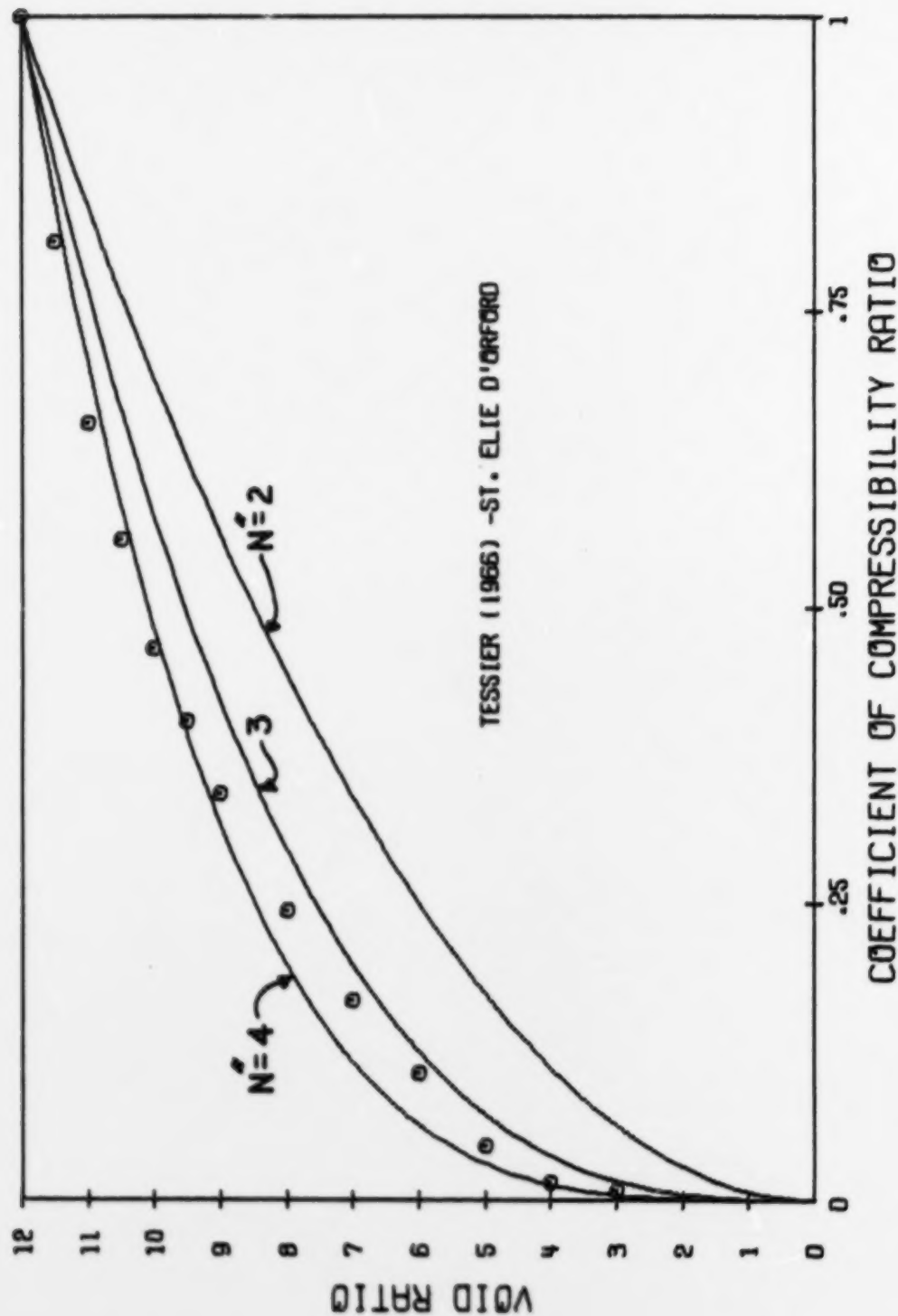


Figure 5-13: Fit of Tessier's (1966) - St. Elie d'Orford data to proposed compressibility function.

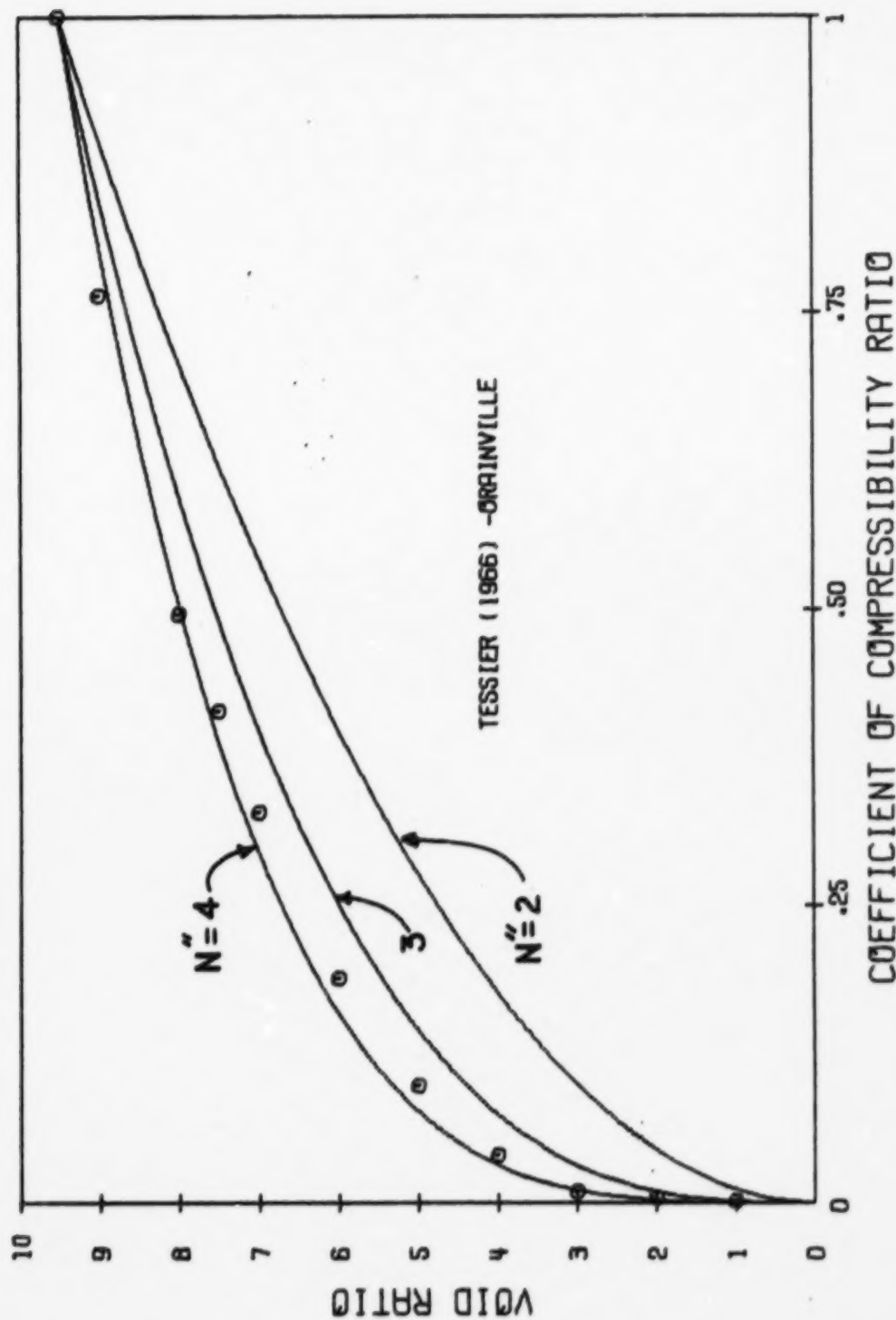


Figure 5-14: Fit of Tessier's (1966) Orainville data to proposed compressibility function.

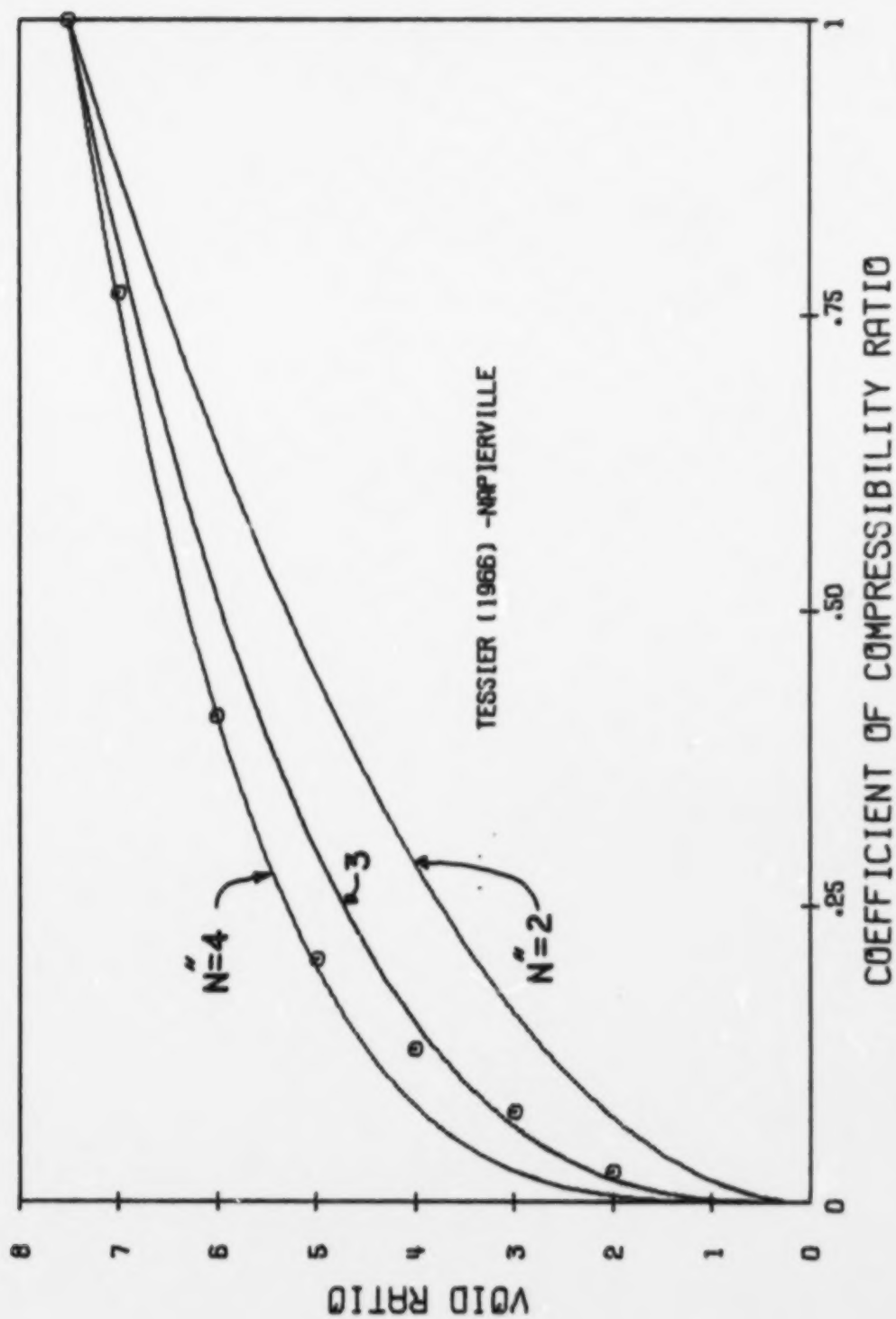


Figure 5-15: Fit of Tessier's (1966) Napierville data to proposed compressibility function.

tion 5-22 can be rearranged to give

$$d\bar{e} = \frac{e_o^{N''}}{a_{v_o}} \frac{de}{e^{N''}} \quad (5-23)$$

which when integrated gives

$$\bar{e}_1 - \bar{e}_o = \Delta p = \frac{e_o^{N''}}{a_{v_o}(N-1)} \frac{e_1^{N''-1} - e_o^{N''-1}}{e_1^{N''-1} e_o^{N''-1}} \quad (5-24)$$

Expanding and solving for the void ratio change gives

$$e_o - e_1 = (N''-1)a_{v_o} \frac{e_1^{N''}}{(e_o^{N''-2} + e_o^{N''-3}e_1 + \dots + e_1^{N''-2})} \Delta p \quad (5-25)$$

which has the general form

$$e_o - e_1 = \bar{a}_v \Delta p \quad (5-26)$$

where $\bar{a}_v$ represents the average compressibility over the entire load range. The coefficient $\bar{a}_v$ depends on the initial and final void ratios, and the value of N'' .

For $N'' = 2$

$$\bar{a}_v = a_{v_o} C_{Av} = a_{v_o} \frac{e_1}{e_o} \quad ; \quad (5-27)$$

for $N'' = 3$

$$\bar{a}_v = a_{v_o} C_{Av} = a_{v_o} \frac{2 e_1^2}{e_o (e_o + e_1)} \quad ; \quad (5-28)$$

while for $N'' = 4$

$$\bar{a}_v = a_{v_0} \quad C_{Av} = a_{v_0} \frac{3 e_1^3}{e_0 (e_0^2 + e_0 e_1 + e_1^2)} \quad (5-29)$$

A plot of void ratio versus C_{Av} is shown in Figure 5-16 for $N'' = 2, 3$, and 4. The function is linear for $N'' = 2$, while for $N'' = 3$ and 4 it is increasingly non-linear. The $N'' = 3$ curve is the function suggested by Figures 5-10 to 5-15.

In a finite difference analysis, each time increment will involve very small changes in the void ratio. The compressibility over any time interval can be assumed constant, and established in such an interval from equation 5-21.

Variations in the Pore Pressure Dissipation Equation Coefficient

The pore pressure dissipation equation has a coefficient which is somewhat similar to the coefficient of consolidation from classical theory. In general, the dissipation equation coefficient will vary with the degree of consolidation, but if the assumptions of the classical theory are made the two coefficients are identical. In the classical theory, the coefficient of consolidation is assumed to be a constant and is expressed as

$$C_v = \frac{k(1 + e)}{\partial_v \gamma_w} \quad (5-30)$$

where

- k = permeability (saturation = 100 percent)
- a_v = compressibility (assumed constant)
- γ_w = unit weight of water
- e = void ratio .

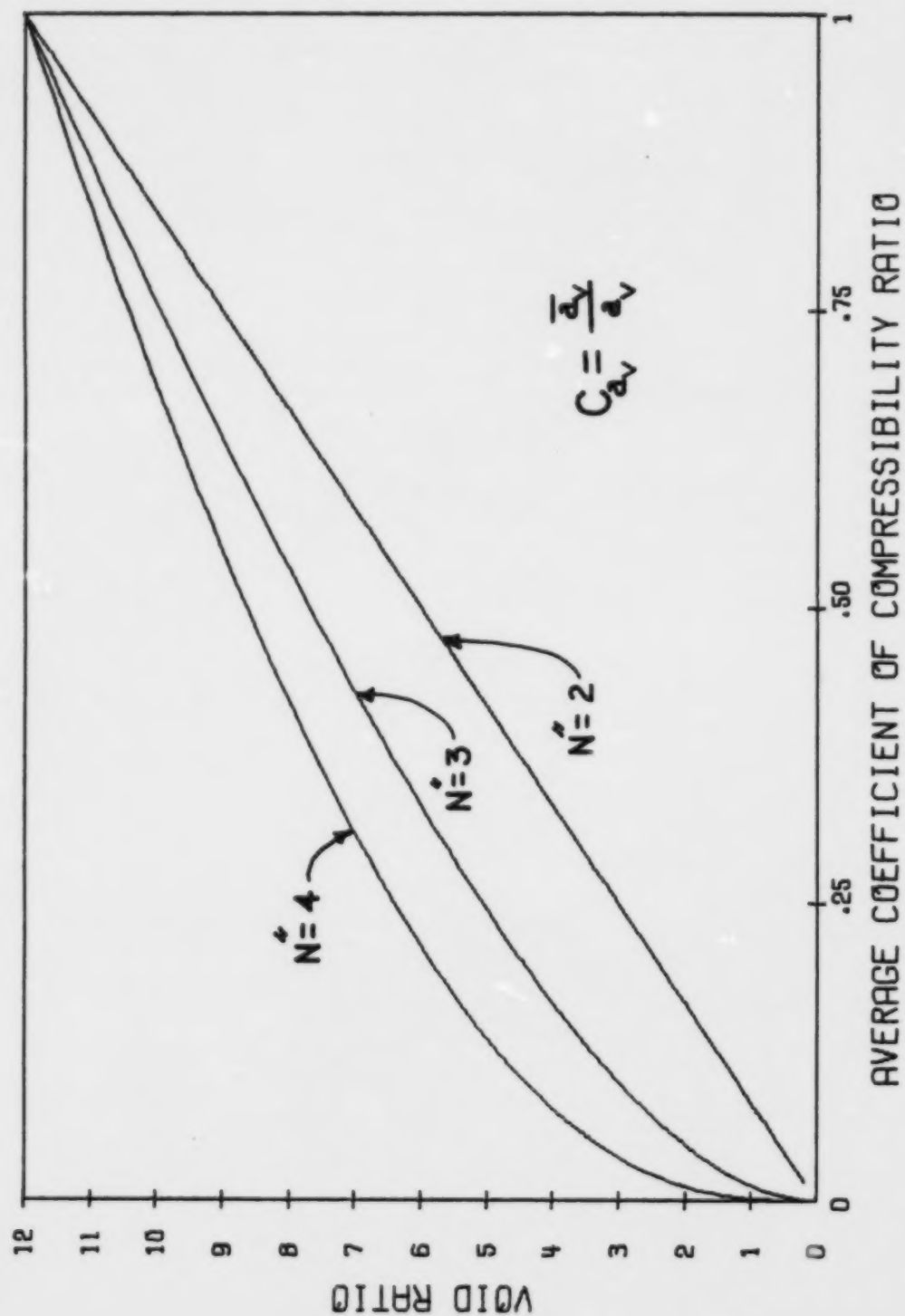


Figure 5-16: Variation of compressibility ratio with void ratio.

Although the compressibility is nearly constant, permeability and void ratio vary enough so that the average rather than the initial values are used.

In the derivation of the continuity equation for this study, the same general terms, as well as additional ones, arise and form a term which is analogous to the coefficient of consolidation. This term is

$$\frac{\bar{K}(1+e)}{\bar{\alpha}_v \gamma_w} \frac{(1+e_o)^e}{(1+e)^2} \quad (5-31)$$

where $\bar{K}$ is the transmissibility. Substituting the equations relating compressibility and permeability to the void ratio yields

$$\frac{K_o(1+e_o)}{\gamma_w \bar{\alpha}_{v_o}} s_e^{N_s} \frac{e_o^{N'}}{e_o^{N'}} \left(\frac{1+e_o}{1+e} \right)^2 \left(\frac{e_o^{N'}}{e^{N'}} \right) \quad (5-32)$$

or

$$\frac{K_o(1+e_o)}{\gamma_w \bar{\alpha}_{v_o}} s_e^{N_s} \frac{e_o^N}{e_o^N} \left(\frac{1+e_o}{1+e} \right)^2 \quad (5-33)$$

The nature of this relationship depends on the relative changes in the compressibility and permeability. Davis and Raymond (1965) assume for clay that the "decrease in permeability is proportional to the decrease in compressibility" or, in other words, the consolidation coefficient is constant for their material. Lea (1958) says that the coefficient of consolidation for a peat sample may vary by as much as a factor of 100, but gives no data for this. Lea and Brawner (1963) report a decrease of the compressibility by a factor of five to 100. They do not show the coefficient of consolida-

tion-void ratio function, but by matching data from their void ratio versus pressure graphs and C_v versus pressure graphs, the curves shown on Figure 5-17 are produced. The figure indicates a linear variation of the coefficient of consolidation with void ratio, but to obtain the large changes quoted by Lea, and Lea and Brawner a second or even third order variation might be necessary.

Figure 5-18 shows the function

$$\frac{C_v}{C_{v_o}} = \left(\frac{e}{e_o}\right)^N \frac{(1 + e_o)^2}{(1 + e)^2} \quad (5-34)$$

plotted against void ratio for a base void ratio of $e_o = 14$. For N equal to unity, the function increases with decreasing void ratio, the opposite of the needed relationship. The second curve, $N = 2$, behaves correctly, but changes in the coefficient are small in the upper void ratio values and large in the lower ones, also undesirable. A nearly linear function results from the third curve, while the fourth ($N = 4$) and fifth ($N = 5$) curves show increasing non-linearity in the desired direction. The curves with $N = 3, 4$, or 5 provide functions which will describe the consolidation coefficient void ratio relationship correctly. If the permeability is a fifth, sixth or seventh order function of the void ratio, while the compressibility is a second, third or fourth order one, then N or $N' - N''$ can easily be third, fourth or fifth order functions, as Figure 5-18 indicates is needed. However, the limited data and lack of identification for the different peats limit the validity of the analysis. The coefficient will be assumed to be governed by equation 5-34.

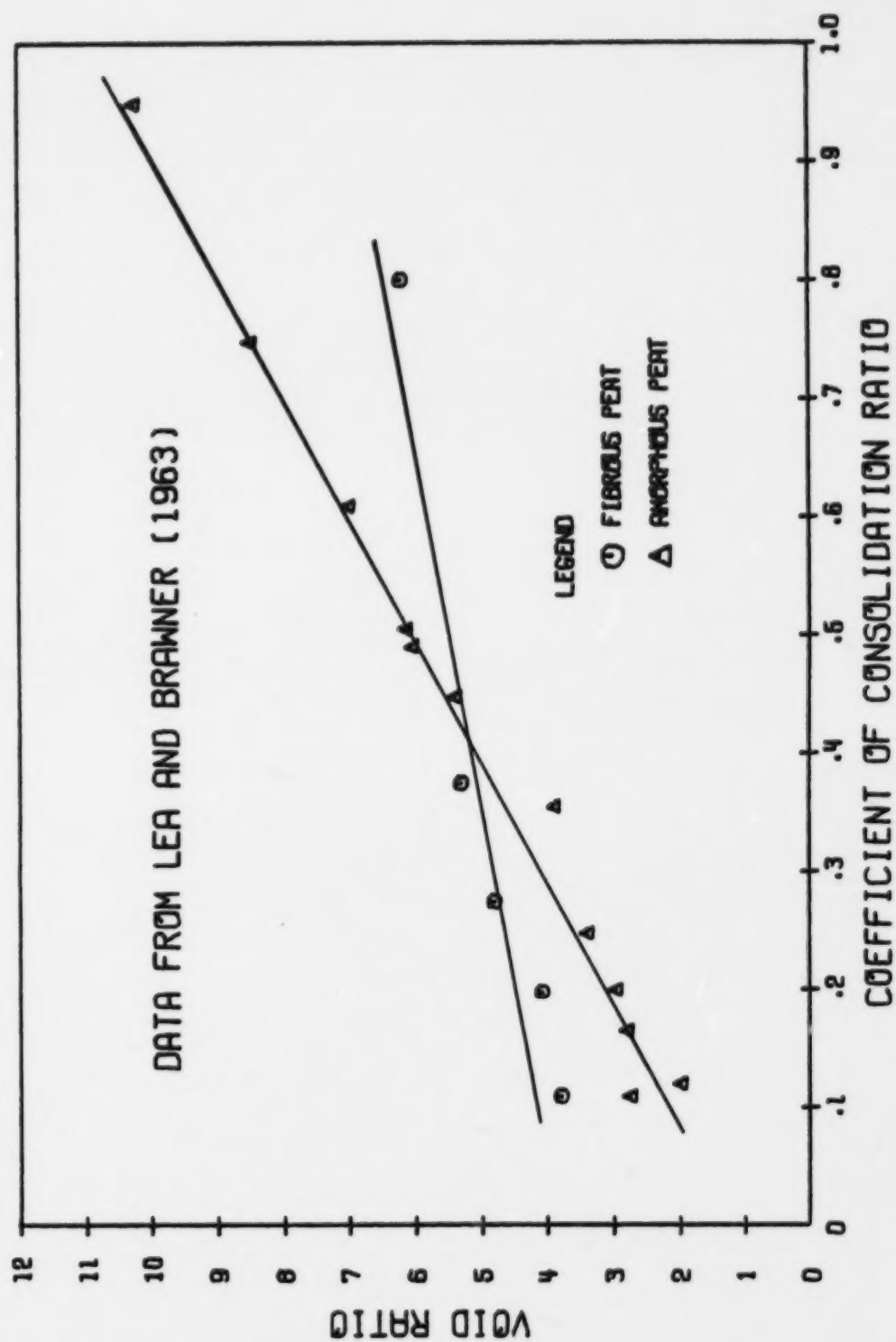


Figure 5-17: Coefficient of consolidation vs void ratio for peat.

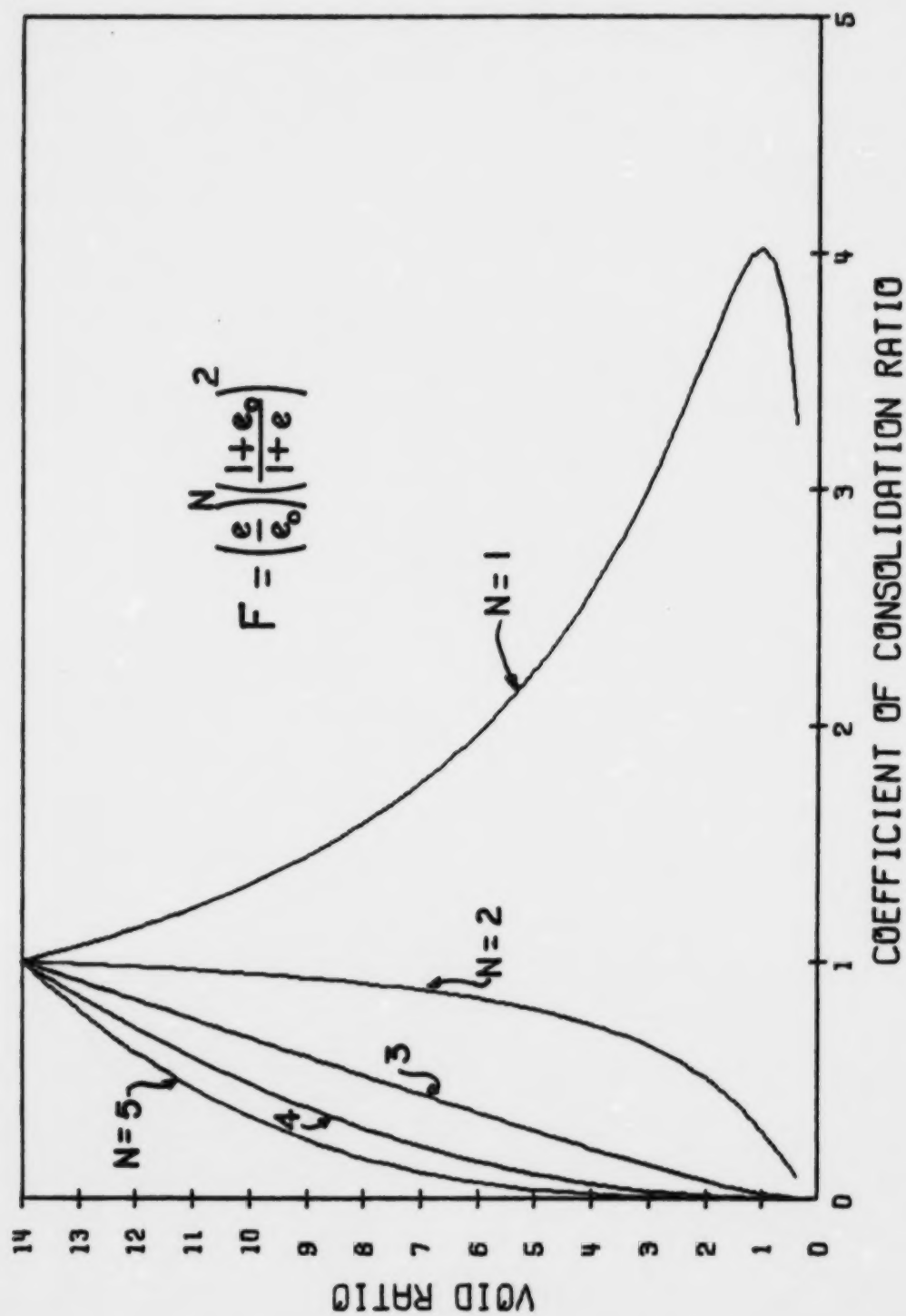


Figure 5-18: Coefficient of consolidation ratio-void ratio functions.

Multi-phase Pore Fluid

A second pore fluid, which is dissimilar from the first, such as a gas with a liquid, produces several complications not present in the single phase system. Because of the gas, the entire system, solids and fluids, is instantaneously compressible, the pore fluid is compressible, and the distribution of stresses among the three phases must be considered. In addition to this, the flow of the fluid is altered, as discussed in the section on transmissibility. The problems of stress distribution and compressibility can be approached in one of two ways; either micro- or macroscopic. A brief outline of the microscopic approach is presented, but, in general, because of its simplicity and acceptable accuracy, a macroscopic approach is taken.

Schuurman (1966) gives a thorough analysis of the liquid-gas compressibility problem. Using Boyle's, Gay-Lussac's and Henry's laws, he analyzes the equilibrium conditions of a gas bubble in a fluid and relates this to the overall compressibility of the liquid-gas mixture. Assuming no flow, constant temperature, and constant liquid to gas ratio (by weight), he arrives at the compressibility relationship

$$C_m = \frac{-1}{V_g + V_f} \left[\frac{V_{go} + V_{do}}{(V_g + V_d)^2} P_{go} + \frac{2q}{3r_o} \frac{1}{V_g} \left[\frac{V_{go}}{V_g} \right] \right]^{1/3} \quad (5-35)$$

where

- C_m = compressibility of mixture
- V_g = volume of gas
- V_f = volume of fluid
- P_{go} = initial gas pressure (absolute)

- V_d = volume of dissolved gas
 q = surface tension of liquid
 r_o = radius of gas bubbles .

This development includes the effects of the surface tension of the fluid. In a fine-grained material such as clay, this produces a relatively large term that cannot be neglected; on the other hand, in a large-grained material such as sand, the effect is small and need not be considered.

Bishop and Eldin (1950) and Bishop (1957) present a compressibility relationship for unsaturated sand which is

$$C_m = (1 - S_o - h_c S_o) \frac{P_{fo}}{(P_f)^2} \quad (5-36)$$

where

- S_o = initial saturation
 h_c = Henry's solubility coefficient
 P_{fo} = initial pore pressure (absolute)
 P_f = pore pressure (absolute) .

This is the result of application of the ideal gas laws to the phase diagram shown in Figure 5-19. Assuming that the load is applied instantaneously, no drainage occurs and the temperature remains constant, the total amount of gas present will not change. If the initial volume of voids is V_v and saturation is S_o , and if h_c is Henry's solubility coefficient, then the initial volume of air is

$$(1 - S_o) V_v \quad (5-37)$$

and the initial dissolved air is

$$h_c S_o V_v \quad . \quad (5-38)$$

At the initial pressure, P_{fo} , the total volume of gas is

$$V_{go} = V_o (1 - S_o - h_c S_o) \quad . \quad (5-39)$$

When the pressure is changed to P_f , the total volume is

$$V_v (1 - S_o - h_c S_o) \frac{P_{fo}}{P_f} \quad . \quad (5-40)$$

Since the amount of gas dissolved in the liquid does not change, the total free volume is

$$V_g = V_v \left((1 - S_o - h_c S_o) \frac{P_{fo}}{P_f} - h_c S_o \right) \quad . \quad (5-41)$$

The change in free volume is

$$V_g - V_{go} = V_v (1 - S_o - h_c S_o) \left(\frac{P_{fo}}{P_f} - 1 \right) \quad . \quad (5-42)$$

If compressibility is defined as the negative of the rate of change of volume with respect to the change in fluid pressure divided by the volume, and since $P_f = P_g$, equation 5-42 will yield the same function as equation 5-36.

The immediate compressibility of the physical system where the solid

particles form a matrix which has some structural strength can also be developed using the ideal gas laws and Figure 5-19. Since the amount of gas present before and after loading is equal, the corresponding volumes can be related using the relationship

$$(P_a + U_n)(V_{g1} - h_c V_{w1}) = (P_a + U_n + \Delta p)(V_{g2} + h_c V_{w1}) \quad (5-43)$$

where

- P_a = atmospheric pressure (absolute)
- U_n = hydrostatic pressure prior to loading
- Δp = increase in load .

The decrease in fluid volume produces an increase in the effective stress, and the volume change and stresses are related by

$$\frac{V_{g1} - V_{g2}}{V_{Total}} = \frac{\Delta e}{1 + e} = a_v \frac{(\Delta p - U_2)}{1 + e_o} \quad (5-44)$$

where

- Δe = void ratio change
- e_o = initial void ratio
- U_2 = excess pressure after loading
- a_v = compressibility
- Δp = increase in load .

By using equations 5-43 and 5-44, the fluid pressure after loading can be found in terms of the matrix properties and saturation to be

$$u_2 = \frac{1}{2} \left[\left[\Delta p - (p_d + u_h) - (v_{g_1} + n_c v_{w_1}) \left(\frac{1+e}{\partial v} \right) \right] + \left[\Delta p - (p_d + u_h) - (v_{g_1} + n_c v_{w_1}) \left(\frac{1+e}{\partial v} \right) \right]^2 + 4(p_d + u_h) \Delta p \right]^{\frac{1}{2}} \quad (5-45)$$

The total change in load is equal to the increase in fluid pressure plus the increase in effective pressure, or

$$\Delta p = u_2 + \frac{v_{g_1} - v_{g_2}}{1 + e_0} \frac{1 + e_0}{\partial v} \quad (5-46)$$

If the gas volume is reduced to zero under the increase in load, then the initial effective stress increase will equal the last term in equation 5-46, and the fluid excess pressure will increase by an amount equal to the remainder of the load change.

The effect of partial saturation upon the distribution of the stresses has been investigated by Bishop (1960a, b), Bishop and Bjerrum (1960), Aitchison (1960), Donald (1961), Jennings and Burland (1962), Bishop and Bligh (1963), and Bishop et al (1964). In general, effective stress in a partially saturated material may be expressed as

$$\bar{\sigma}_T = \bar{\sigma} + u_g + \alpha (u_g - u_f) \quad (5-47)$$

where

u_g = excess gas pressure

U_f = excess fluid pressure

$\bar{\sigma}_T$ = total stress

α = parameter that depends primarily on the degree of saturation, but also on structure, stresses and how the present saturation state was reached.

The parameter α is difficult to evaluate, and will differ for each type of material. However, Donald suggests as an acceptable approximation

$$\alpha = S \quad (5-48)$$

The effect of the surface tension may be neglected if the pore radius of the material is greater than that of a clay or fine silt. Assuming that this effect can be neglected, the initial reaction of the material to the load increase can be described by the simple relationships based on the phase diagram of Figure 5-19.

Saturation-Time

When the initial load is applied to the system, the volume of gas is decreased and the degree of saturation thus increases. If the loading is instantaneous, no drainage will occur and the ideal gas laws can be applied as they were for equations 5-36 to 5-46. However, during the consolidation process the pressure in the fluid will decrease and the gas will tend to expand and decrease the degree of saturation. A closed form expression for the saturation variation with degree of consolidation, that is, with changing pressure and drainage, is not practical. The variation of the saturation can be estimated, using the finite difference approach. At any given time, t_1 , the saturation, pore pressure, and void ratio are known,

while at time $t_1 + \Delta t$, there is a new pressure and void ratio. The saturation will change with time due to the flow of gas and liquid, as well as the decrease in pressure. If it is assumed that the fluid flow is homogeneous, the outflow has the same consistency as that remaining fluid, then the new saturation can be estimated from

$$S_{t+\Delta t} = \frac{S_t}{1 + \left(\frac{P_{ft}}{P_{t+\Delta t}} \right) (1 - S_t + n_c S_t)} \quad (5-49)$$

Summary

The variations of some of the material parameters during the consolidation process are discussed in this chapter. Assuming that Darcy's law is valid, the variation of permeability with geometry is discussed and compared with actual data. An excellent fit is found between the data and the proposed permeability-void ratio relationship given by equation 5-11. Transmissibility's variation with saturation is assumed to follow the empirical function of Brutsaert given by equation 5-16. The changes in compressibility with void ratio are found to fit the function proposed in equation 5-21 for the available data. The transmissibility and compressibility functions are used to investigate changes in the coefficient of the pore pressure dissipation equation, and their interaction is found to agree with the very limited data. Although there is little available data, functions are found for permeability, transmissibility, compressibility, and the coefficient to the dissipation equation which tend to substantiate each other. The effect of multi-phase fluid, particularly liquid and gas, upon the behavior of the material is studied, and expressions for the material's instantaneous compressibility and load distribution are developed,

as well as a function for the saturation variation between any two time intervals.

CHAPTER VI

PORE PRESSURE DISSIPATION AND CREEP

Introduction

The final working model that will incorporate the results of Chapters IV and V is developed in this chapter. The relationships developed in Chapter V must first be substituted into equation 4-76 to simplify the function. The effects of the different variables on the fluid behavior are then studied. Finally, equation 4-84, the creep function, is solved for various values of the controlling parameters, and the solution to the overall problem is discussed.

Pore Fluid Behavior

The pore fluid model developed in Chapter IV still requires the substitution of the functions from Chapter V to make it a working relationship. With these substitutions, and several additional assumptions, the effect of the variation of several different parameters on pore fluid behavior alone will be investigated.

If the saturation is assumed to be constant with depth, and transmissibility the function of void ratio and saturation shown in equation 5-1, then

$$\begin{aligned}\frac{\partial S}{\partial z} &= 0 \\ \frac{\partial \bar{K}}{\partial z} &= \bar{K}_z \frac{\partial e}{\partial z}\end{aligned}\tag{6-1}$$

Also, if the change in void ratio with depth is assumed to be a function of the change in fluid pressure with depth, then

$$\frac{\partial e}{\partial z} = A' \frac{\partial u}{\partial z}\tag{6-2}$$

where A' is not necessarily constant, but should be positive. Substituting equations 6-1 and 6-2 into 4-75 yields

$$\begin{aligned} \frac{\partial \lambda}{\partial t} = & \frac{(1+e_0)^2}{1+e} \frac{\bar{k}}{\gamma_w \bar{a}_v H^2} \frac{u_0}{\Delta p} \left(\frac{\partial^2 u}{\partial z^2} + \left(\frac{\bar{k}_z}{\bar{k}} - \frac{1}{1+e} \right) A' \left(\frac{\partial u}{\partial z} \right)^2 \right) \\ & + \frac{(1+e)\bar{\gamma}}{s \gamma_w} \gamma_f - \frac{e}{s} \frac{\partial s}{\partial t} \end{aligned} \quad (6-3)$$

The coefficient of the first term on the right hand side of this equation, which for the time being will be called L , can be rearranged if the functions for the permeability and compressibility from Chapter V are substituted yielding,

$$L = \frac{k_0}{\gamma_w \bar{a}_v H^2} S_e^{N_s} \frac{(1+e_0)^2}{1+e} \frac{F(e)}{G(e)} \frac{u_0}{\Delta p}$$

which, upon rearranging, gives

$$L = \frac{k_0(1+e_0)}{\gamma_w \bar{a}_v H^2} \frac{1+e_0}{1+e} S_e^{N_s} \frac{u_0}{\Delta p} \frac{F(e)}{G(e)} \quad (6-4)$$

where $F(e)$ and $G(e)$ represent functions of void ratio. If the compressibility is constant, then c_{a_v} is equal to unity and the first part of the term is the same as that preceding the pore pressure term in the Terzaghi equation. The second part of the term involving the initial and instantaneous void ratios is a result of the finite deflections considered. The saturation effect is accounted for in the third term, while the ratio of the initial pore pressure to the load increase also is a function of saturation, which is discussed in Chapter V. The last term represents the ratio of the change

in transmissibility to the change in compressibility. If equations 5-11 and 5-19 are used, then

$$\frac{F(e)}{G(e)} = \left(\frac{e}{e_0} \right)^N \frac{(1+e_0)}{(1+e)}, \quad (6-5)$$

Thus, the entire coefficient can be expressed as

$$L = \frac{k_o(1+e_0)}{\gamma_w a_v H^2} \frac{(1+e_0)^2}{(1+e)^2} \left(\frac{e}{e_0} \right)^N S_e^{N_2} \frac{U_s}{\Delta p} \quad (6-6)$$

and will vary as shown in Figure 5-18, depending upon the value of N . The coefficient associated with the square of the first derivative of the pore pressure in equation 6-3 can be evaluated using the permeability function from equation 5-11 to give

$$A' \left(\frac{N'}{e} - \frac{2}{1+e} \right) = A'' \quad (6-7)$$

Figure 6-1 shows the variation of the term $\left(\frac{N'}{e} - \frac{2}{1+e} \right)$ with void ratio. For large void ratios the function is nearly linear, while as the void ratio approaches zero the function approaches infinity. The other member of the coefficient, A' , relates void ratio change with depth to pore pressure change with depth, and will decrease with the void ratio and pore pressure, as they decrease with time. Thus, factor A' decreases while the factor $\left(\frac{N'}{e} - \frac{2}{1+e} \right)$ increases. Because there is no information as to the actual behavior of either factor, it will be assumed that their product is constant; or

$$A'' = A' \left(\frac{N'}{e} - \frac{2}{1+e} \right) = \text{constant} \quad (6-8)$$

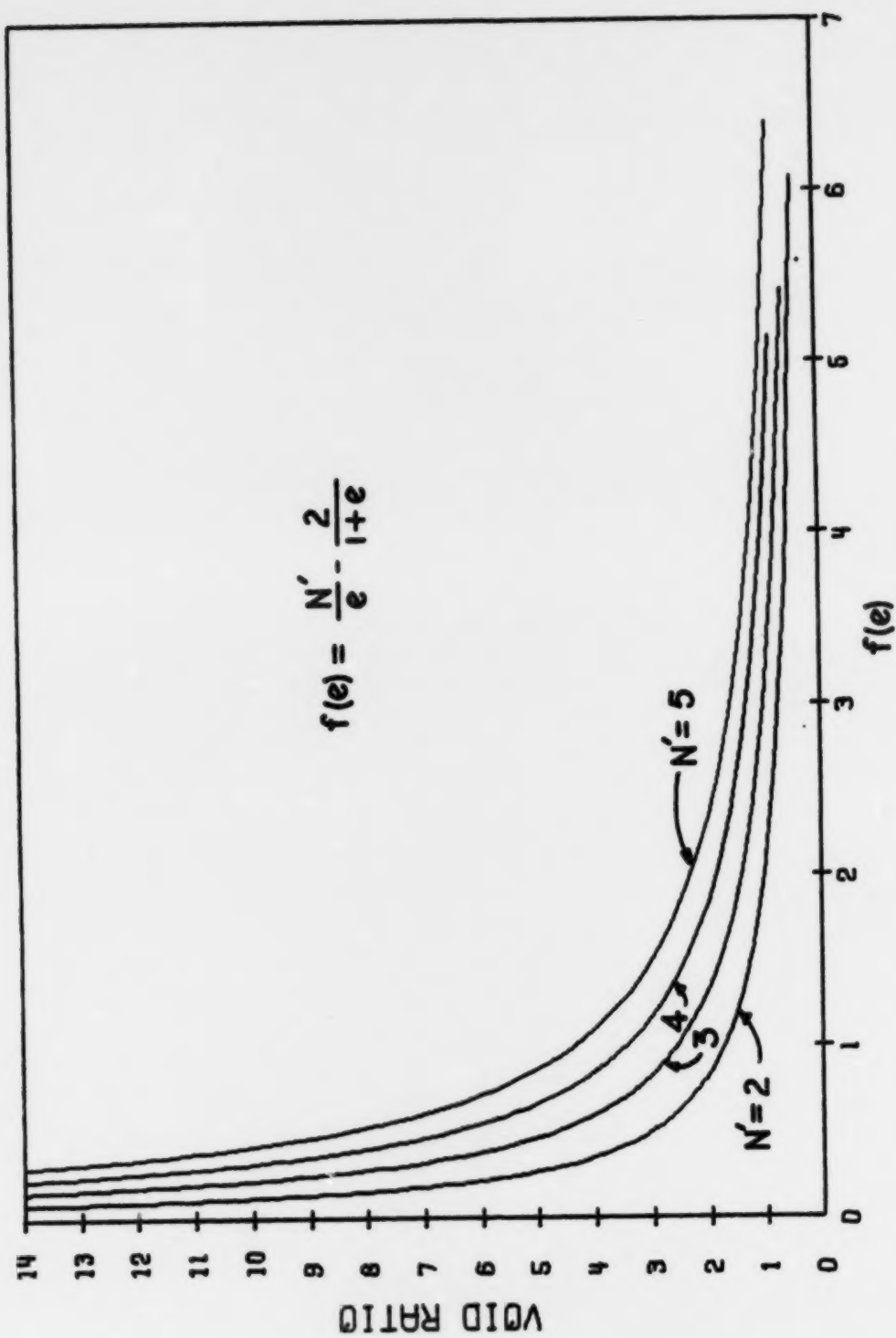


Figure 6-1: Variation of the coefficient $\left(\frac{N'}{e} - \frac{2}{1+e}\right)$.

Dimensionless time is introduced by using the "time factor,"

$$\tau = \frac{k_o(1+e_o)}{\gamma_w \partial_v H^2} t \quad (6-9)$$

Substituting equations 6-6, 6-8 and 6-9 into 4-76 yields

$$\begin{aligned} \frac{\partial \lambda}{\partial \tau} = & s_e^{N_s} \frac{u_o}{\Delta P} \left(\frac{e}{e_o} \right)^N \left(\frac{1+e_o}{1+e} \right)^2 \left(\frac{\partial^2 u}{\partial Z^2} + A'' \left(\frac{\partial u}{\partial Z} \right)^2 \right) \\ & + \frac{\partial_w H^2}{k_o(1+e_o)} \frac{(1+e) \bar{g}}{s \gamma_w} \psi' - \frac{\theta}{s} \frac{\partial s}{\partial \tau} \end{aligned} \quad (6-10)$$

This is the general pore fluid equation; the first term on the right hand side deals with the classical pressure dissipation process while the second involves fluid generation and saturation changes. Equation 6-10 is a non-linear partial differential equation. Such equations are often associated with troublesome convergence and stability problems, but, according to Douglas (1960) and Peaceman (1967), if the lower order term of the equation is bounded, the non-linearity of that term will not affect the convergence. Since A'' is a finite term, and $\partial u / \partial Z$ is bounded on its interior interval, the whole lower order term is bounded, and thus the non-linear term will not affect the convergence. Stability of the solution is assured by taking smaller and smaller increments of the term $\Delta T / \Delta Z^2$ until constant results are obtained. Appendix B contains a detailed discussion of stability and convergence.

Saturated Case

If it is postulated that the material is totally saturated, that A'' is

zero, and that no fluid generation occurs, then equation 6-10 becomes

$$\frac{\partial u}{\partial T} = \left(\frac{c}{e_o} \right)^N \left(\frac{1 + e_o}{1 + e} \right) \frac{\partial^2 u}{\partial Z^2} \quad (6-11)$$

Since creep is neglected, $\lambda = u$ and the function is the same as the classical one except for the variable coefficients.

As can be seen from Figure 5-18, the coefficient $\left(\frac{e}{e_o} \right)^N \left(\frac{1 + e_o}{1 + e} \right)^2$, which will be referred to as Ω , in equation 6-11 increases, for $N > 1$, with the void ratio; or as the degree of consolidation increases, it increases at a rate depending upon the power N of the ratio of the void ratios. The solution to the equation is obtained using simple explicit finite difference methods and the approximations given in Appendix A. At every time interval the value of the coefficient Ω is calculated for each node. The results of these calculations are given as degree of consolidation versus dimensionless time T for N equal to 2, 3, 4, and 5, are shown in Figures 6-2, 6-3, 6-4, 6-5, and 6-6, the corresponding movements being 5, 10, 16.67, 33.5 and 50 percent at the original height. The variations of the coefficient Ω with void ratio for equation 6-11 for the different cases are shown in Figures 6-7 to 6-11.

The time-settlement graphs show that the time required to reach a given state of consolidation increases with the N value and the influence of the value of N increases with the amount of volume change. The graphs of Figures 6-7 through 6-11 show that the coefficient Ω varies considerably with the value of N , and its range increases with the amount of movement. Figure 6-12 shows, as expected, that as the amount of compression increases, the time required to reach equilibrium also increases for the same material and

a given value of N .

The real time involved in the consolidation process considered is not evident from the time-settlement graphs, in which the "time factor," T , is in terms of dimensionless time. An appreciation for the meaning of the dimensionless time can be obtained from equation 6-9 if certain assumptions are made for the characteristics of the materials. A loose milled refuse material may have a void ratio of, say, 13, a permeability of 10^{-3} cm/sec, and a compressibility of $15 \text{ cm}^2/\text{kg}$, while in a dense state the void ratio could be 3, the permeability 5×10^{-6} cm/sec and the compressibility $4 \text{ cm}^2/\text{kg}$. From equation 6-9 the loose material would have corresponding to the unit dimensionless time a real time of 1.04 days, while the corresponding real time for the denser material would be 208 days. In Figure 6-2 the duration of settlements would then be 2.4 days for the loose material, and 476 days, or 1.3 years, for the dense. The times in Figure 6-6 are the same as in Figure 6-2 for $N = 2$; but for $N = 5$, the settlement would take 11 days for the loose, and 2080 days, or 5.7 years, for the dense material. However, a 50 percent volume change of the material is represented in Figure 6-6, and while this may be a common amount for the loose material, it would be uncommon for the dense. Thus, the times of settlement involved in this study might range from about one day to several years for values of the material parameters which seem to be plausible.

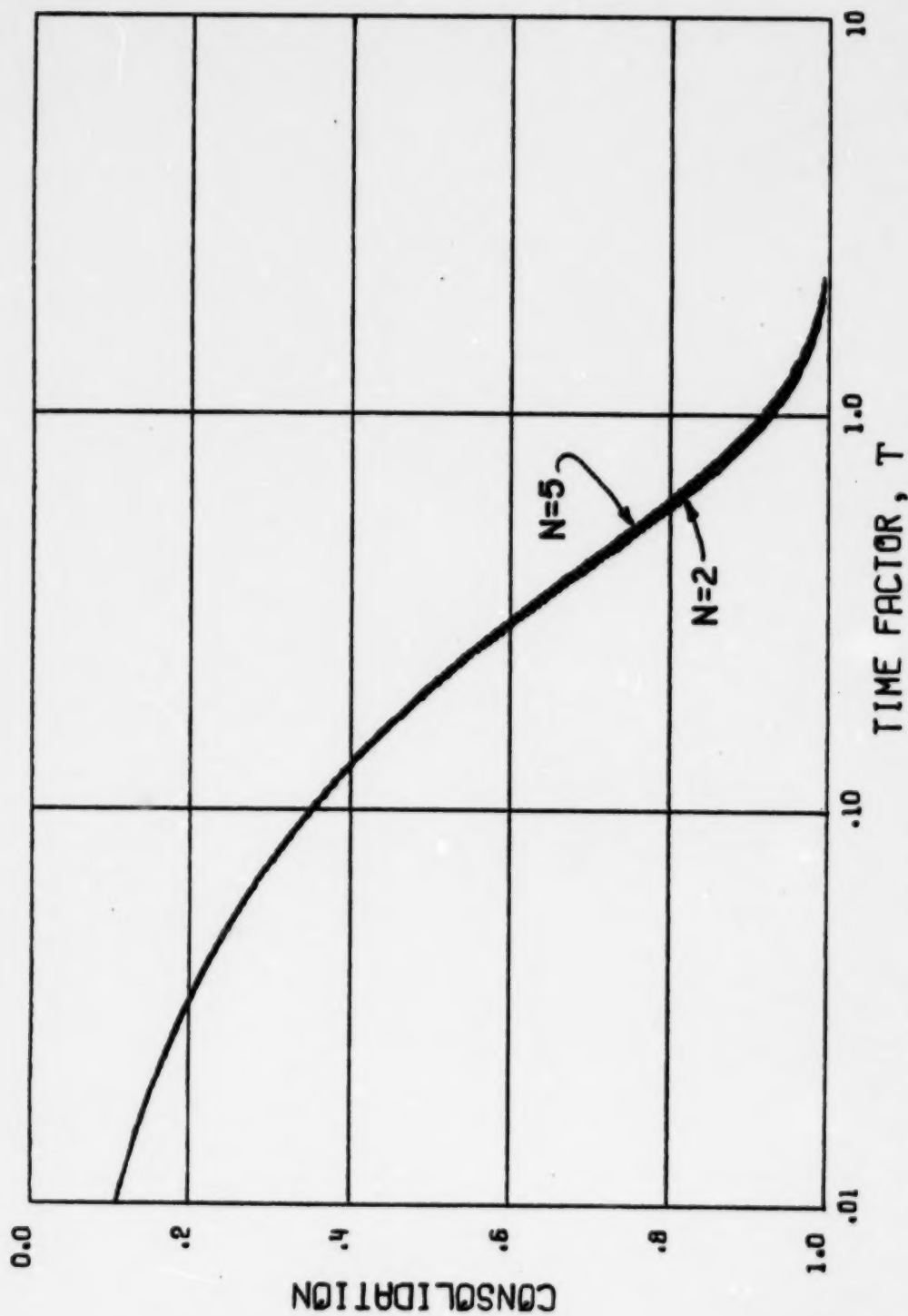


Figure 6-2: Degree of consolidation versus time for saturated material and five percent volume change.

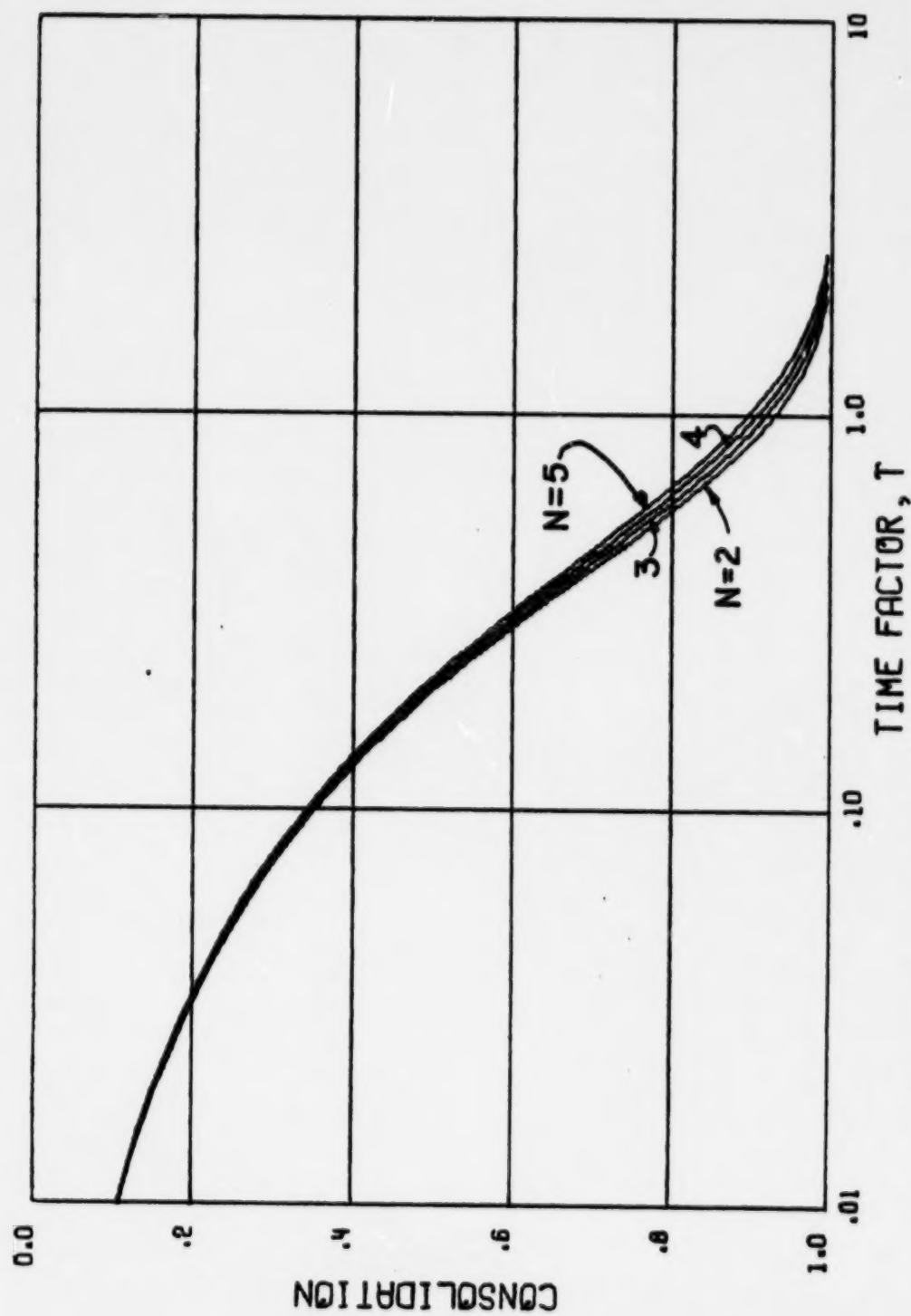


Figure 6-3: Degree of consolidation versus time for saturated material and ten percent volume change.

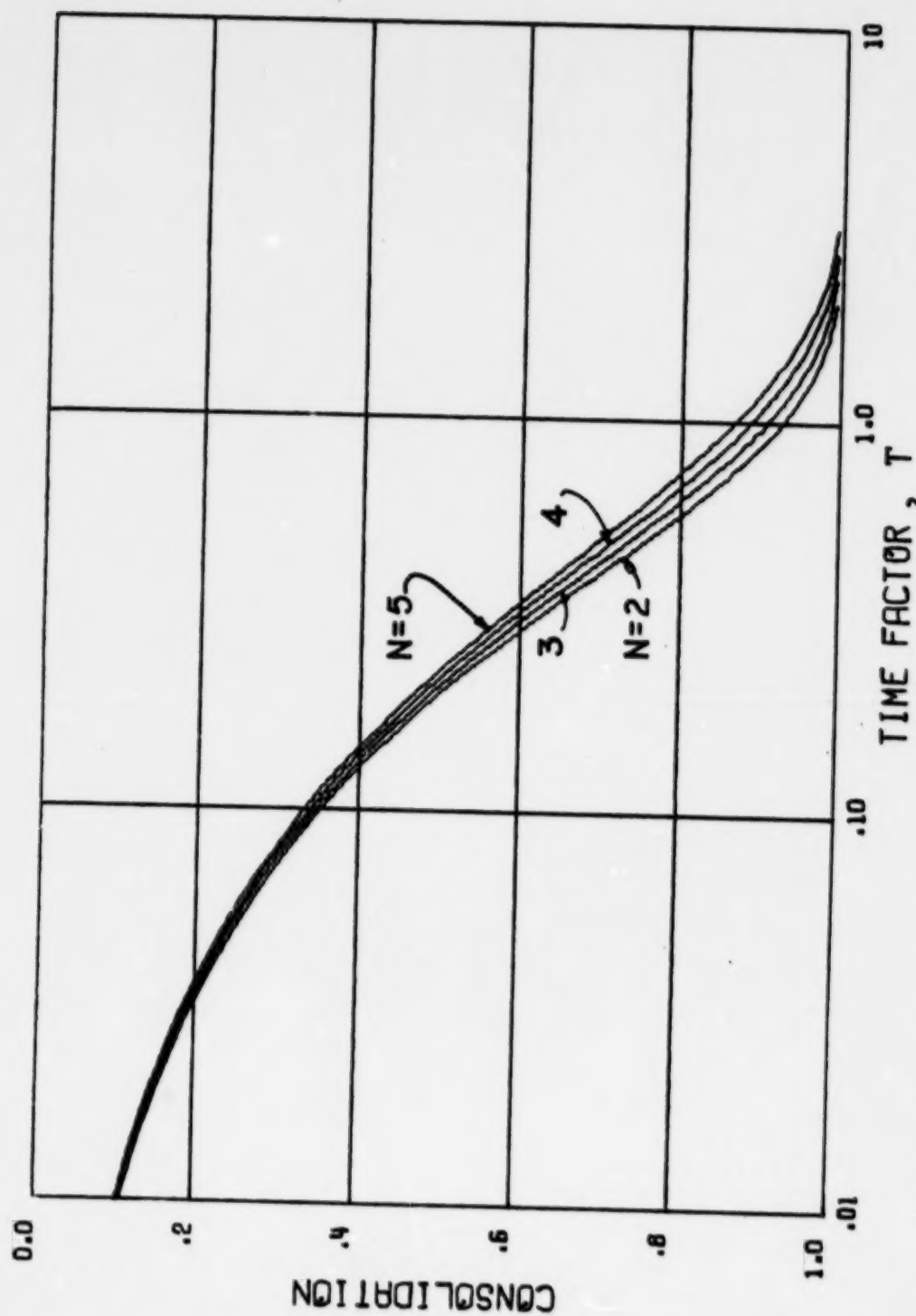


Figure 6-4: Degree of consolidation versus time for saturated material with sixteen and two-thirds percent volume change.

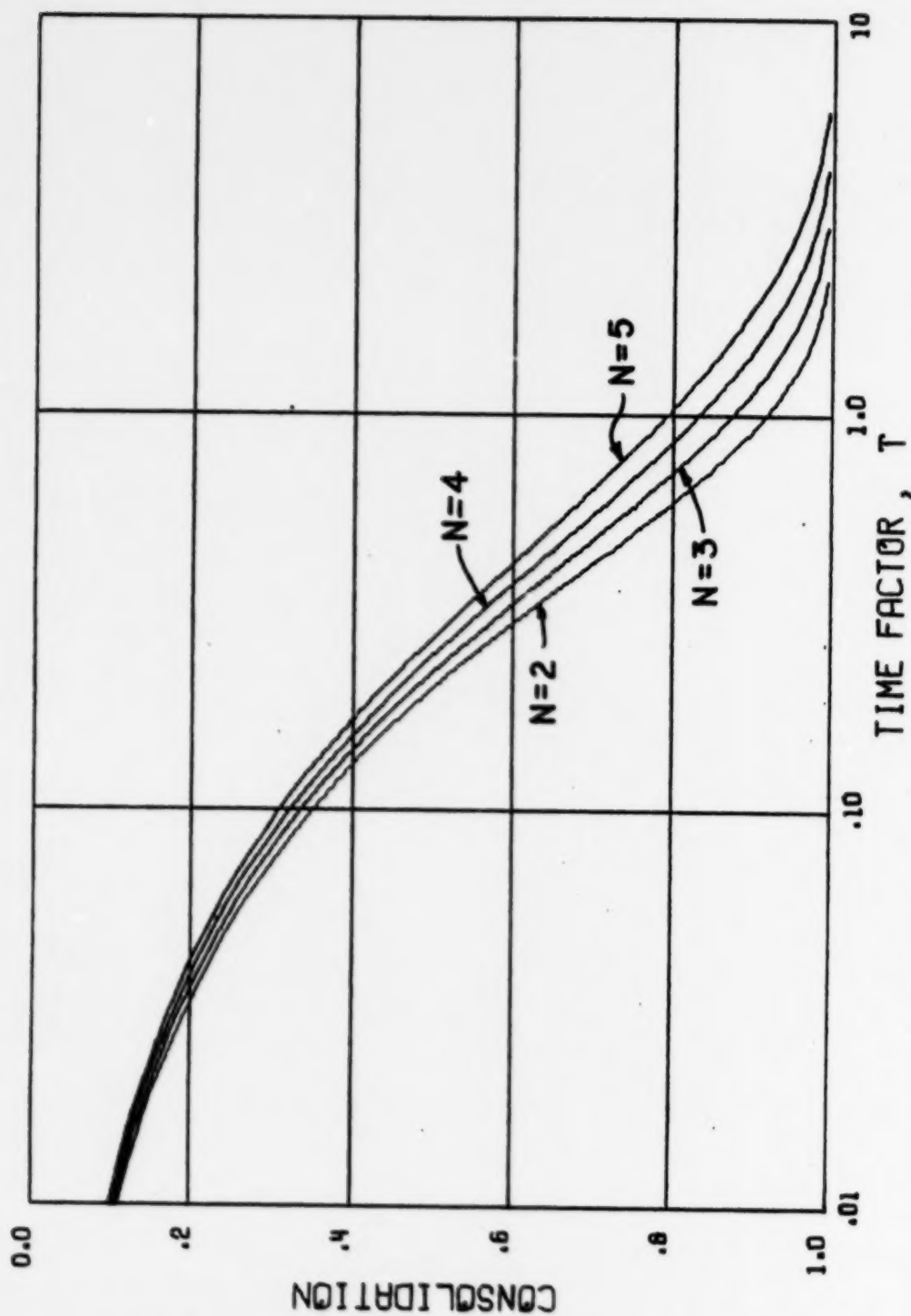


Figure 6-5: Degree of consolidation versus time factor for saturated materials and thirty-three and one third percent volume change.

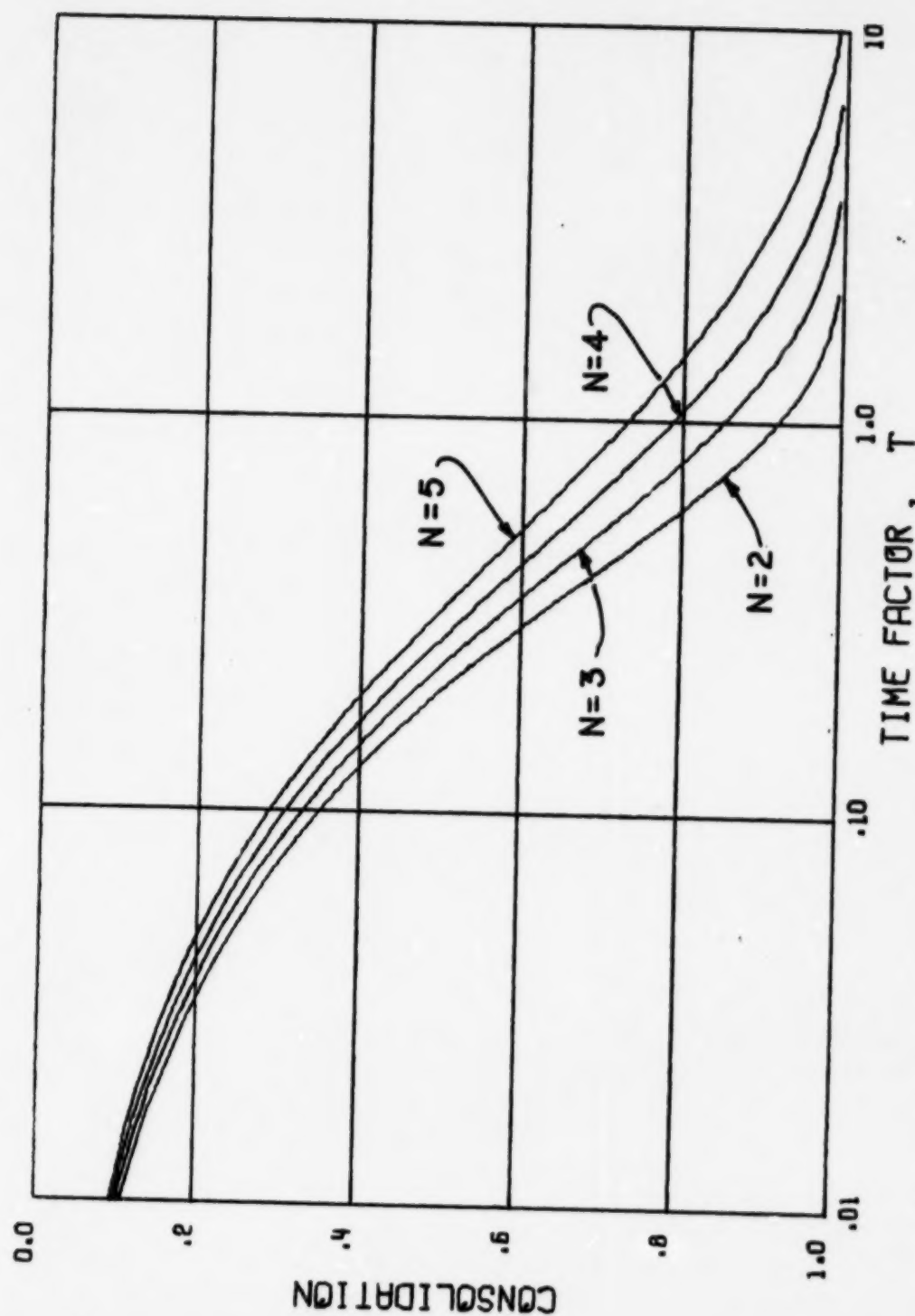


Figure 6-6: Degree of consolidation versus time factor for saturated material and fifty percent volume change.

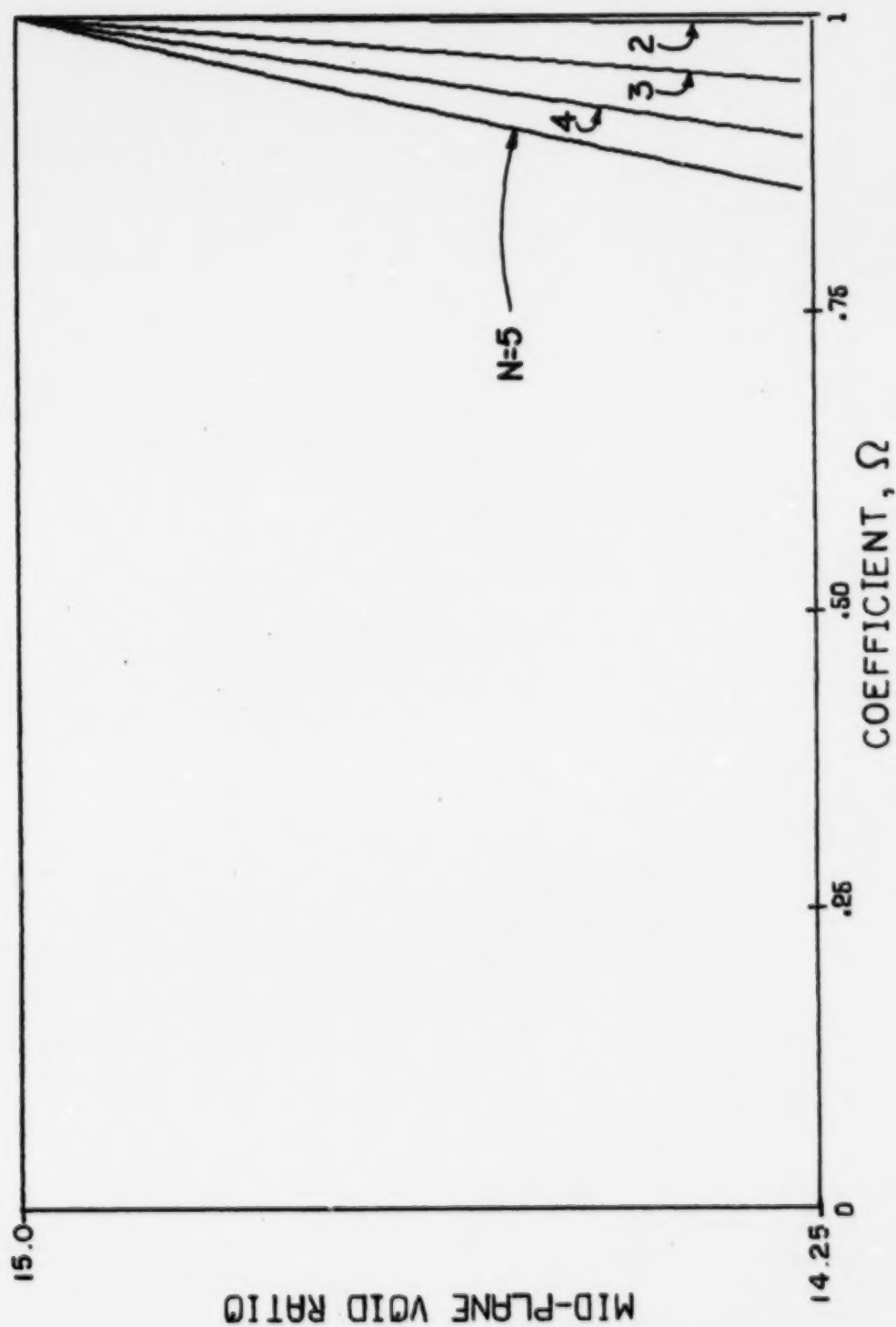


Figure 6-7: Variation of coefficient $\left(\frac{e}{e_0}\right)^N \left(\frac{1+e_0}{1+e}\right)^2$ with mid-plane void ratio for five percent volume change

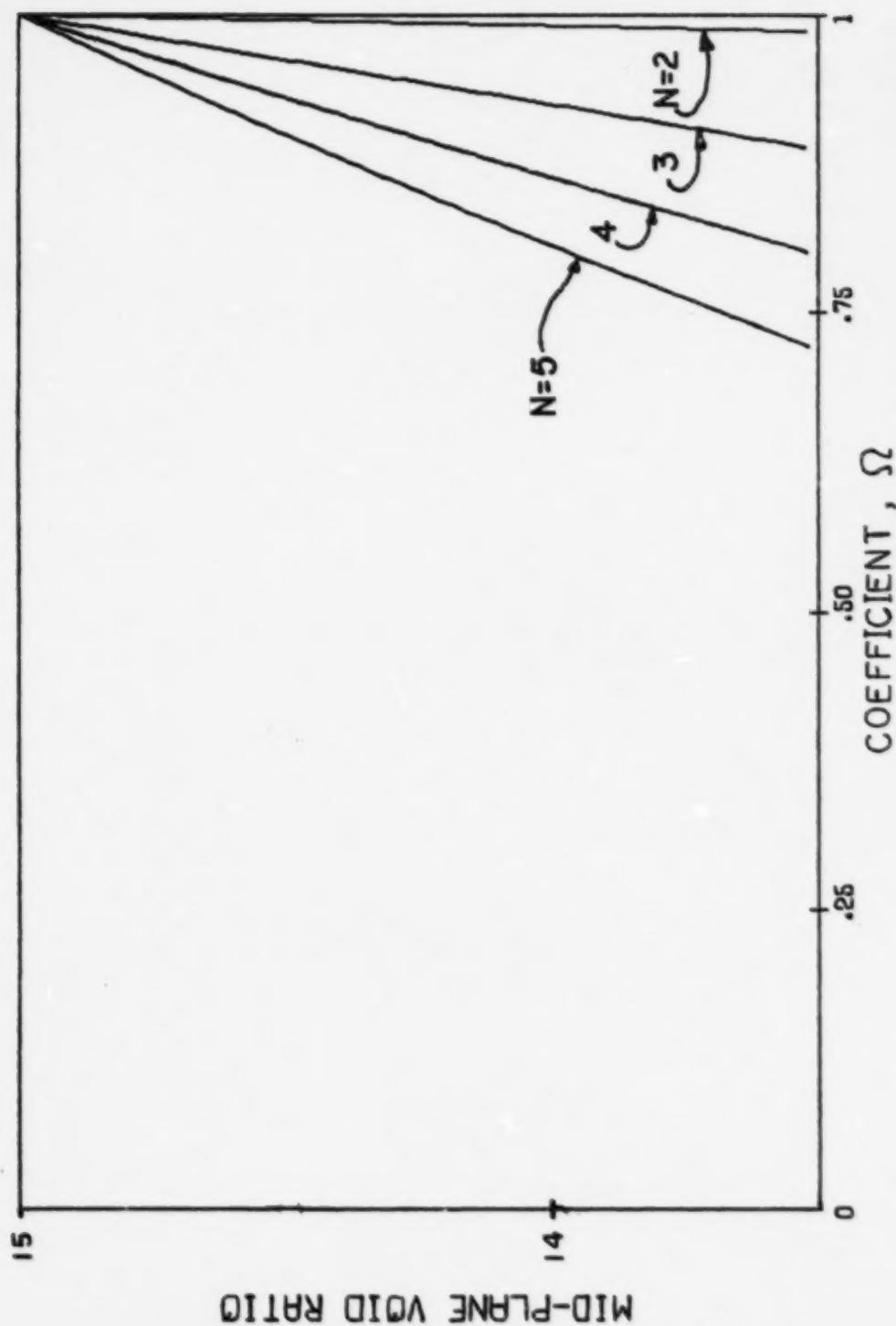


Figure 6-8: Variation of coefficient $\left(\frac{e}{e_0}\right)^N \left(\frac{1+e_0}{1+e}\right)^2$ with mid-plane void ratio for ten percent volume change

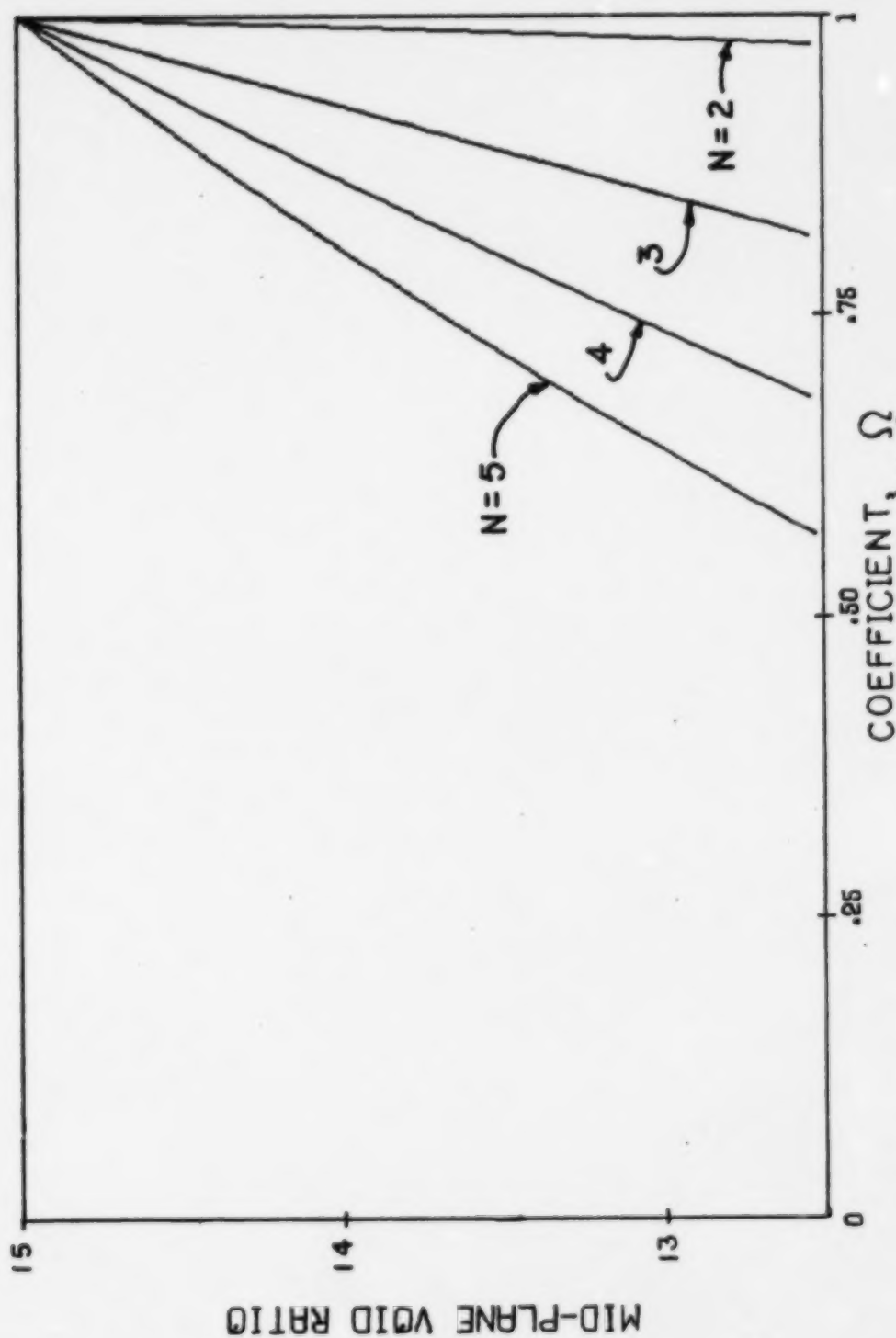


Figure 6-9: Variation of coefficient $\left(\frac{e}{1+e}\right)^N$ with mid-plane void ratio for sixteen and two thirds percent volume change.

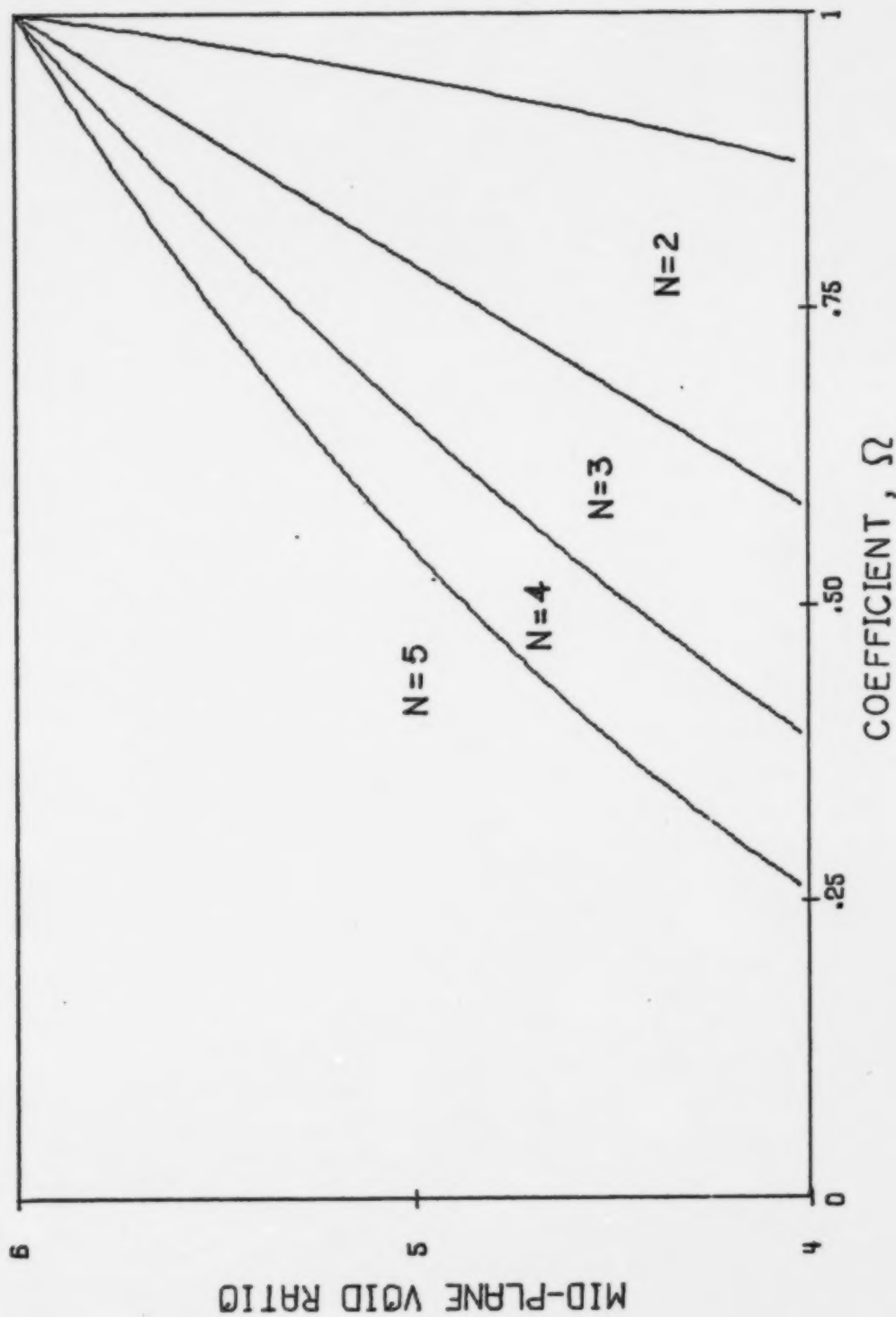


Figure 6-10: Variation of coefficient $(\frac{e}{e_0})^N (1 + \frac{e_0}{1+e})^2$ with mid-plane void ratio for thirty three and one third percent volume change.

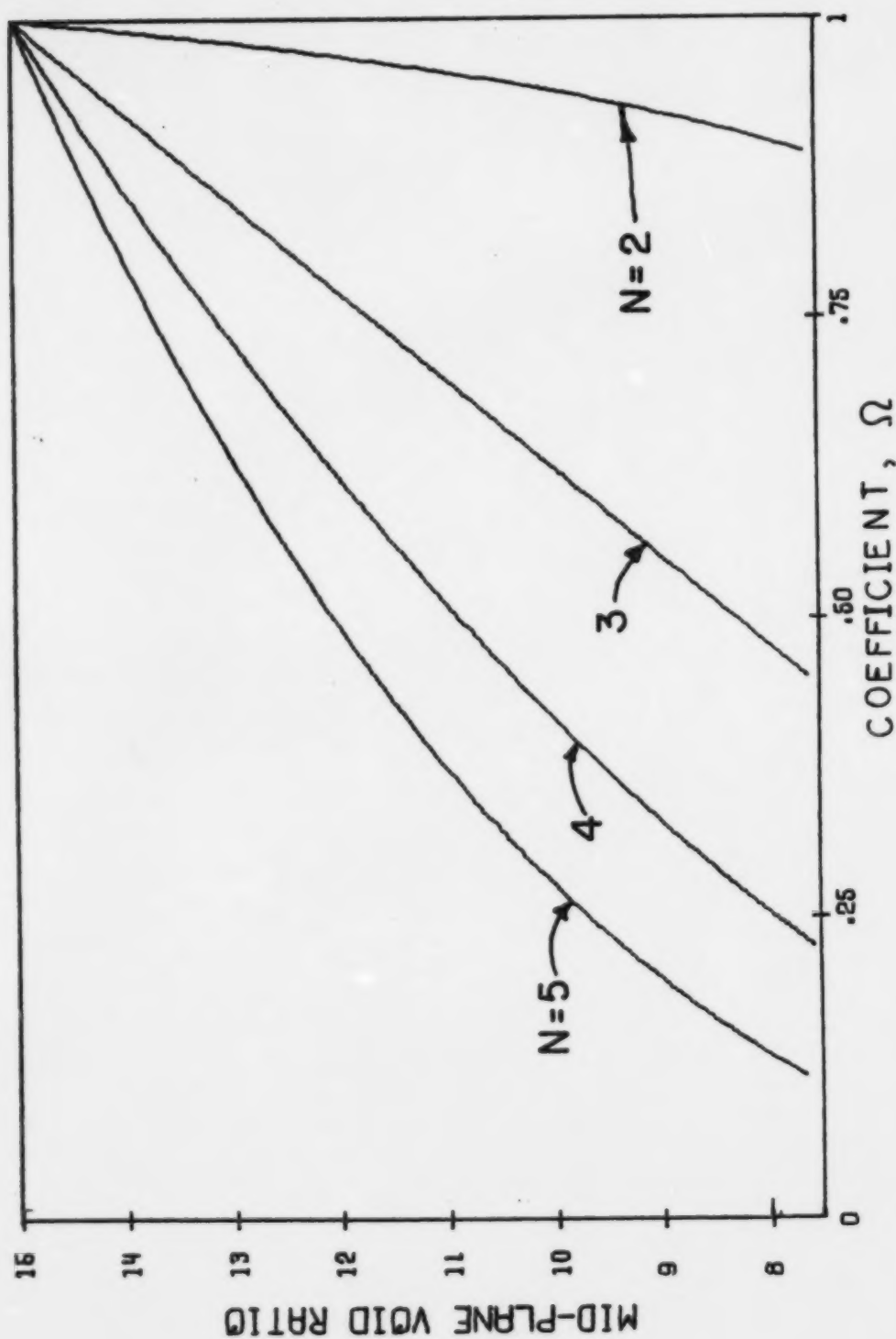


Figure 6-11: Variation of coefficient $(\frac{a}{b_0})^N (1+\frac{e_0}{1+e})^2$ with mid-plane void ratio for fifty percent volume change

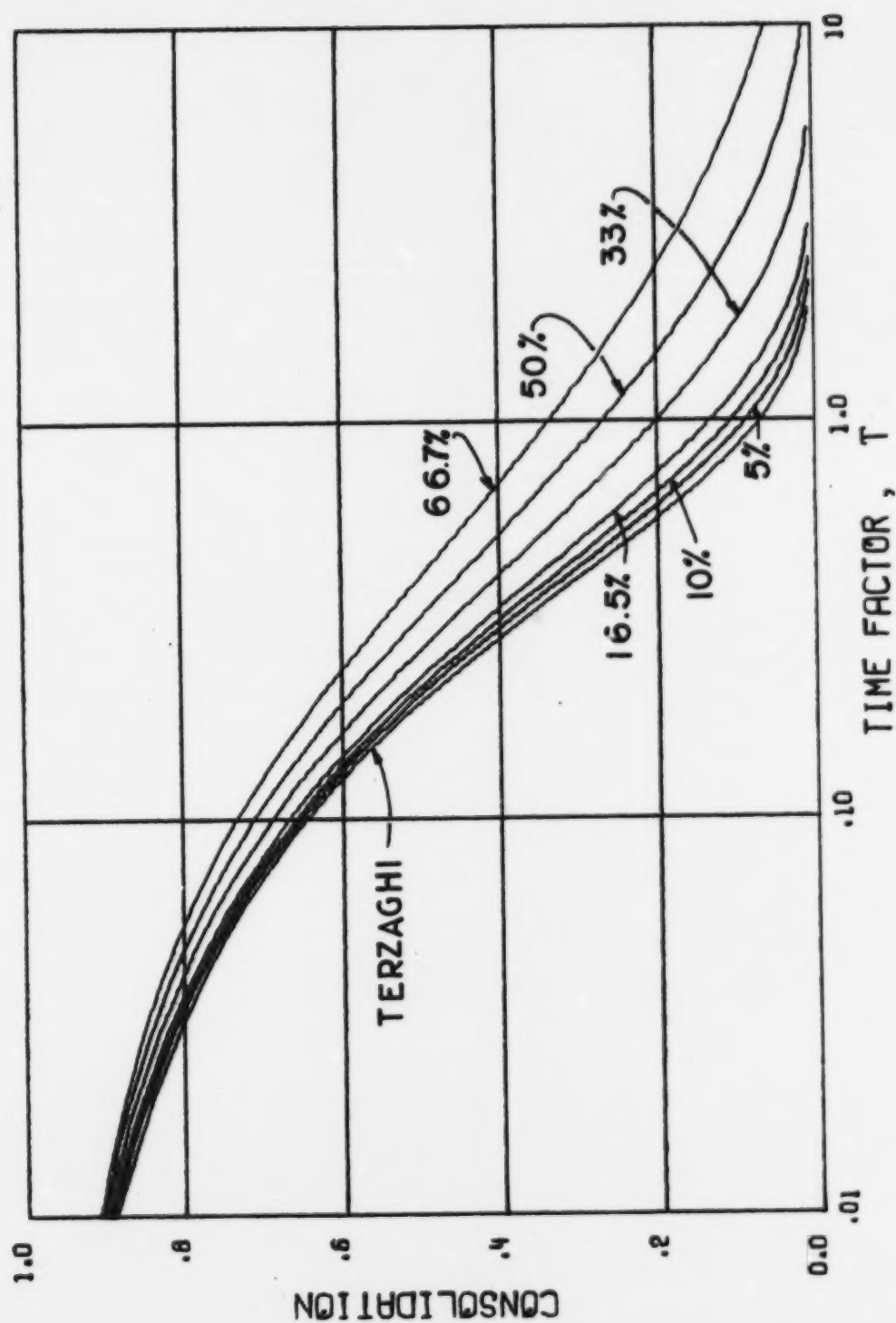


Figure 6-12: Variation in behavior with constant N value ($N=5$) and different percentages of volume change.

If A' is not zero and the material is saturated, then equation 6-10 becomes

$$\frac{\partial u}{\partial \tau} = \left(\frac{e}{e_0} \right)^N \left(\frac{1+e_0}{1+e} \right)^2 \left(\frac{\partial^2 u}{\partial z^2} + A' \left(\frac{\partial u}{\partial z} \right)^2 \right) \quad (6-12)$$

which is a non-linear equation due to the inclusion of a second-degree term in the first partial derivative of the pore pressure. The coefficient A'' is positive when the material is consolidating since $\left(\frac{N'}{e} - \frac{2}{1+e} \right)$ is always positive for $N'' \geq 2$ and A'' is also positive. Figure 6-13 shows the family of settlement-time curves for N equal to 3 and 33 percent movement, while in Figure 6-14 N is 4 and the settlement is 16.67 percent of the original height. For both families of curves, the time of settlement completion increases with the value of the coefficient A' .

Viscous Behavior

The basic creep function for the model is given by equation 4-84, and can be studied independent of the influence of the fluid pressure. If no excess fluid pressures exist, then the viscous part of the model is expressed as

$$\frac{\partial \lambda}{\partial \tau_s} = \frac{1}{\Delta p \phi_c} \sinh(\phi_c \Delta p \lambda) \quad (6-13)$$

which can be solved either by finite difference techniques, or by integration. Integration of 6-13 yields

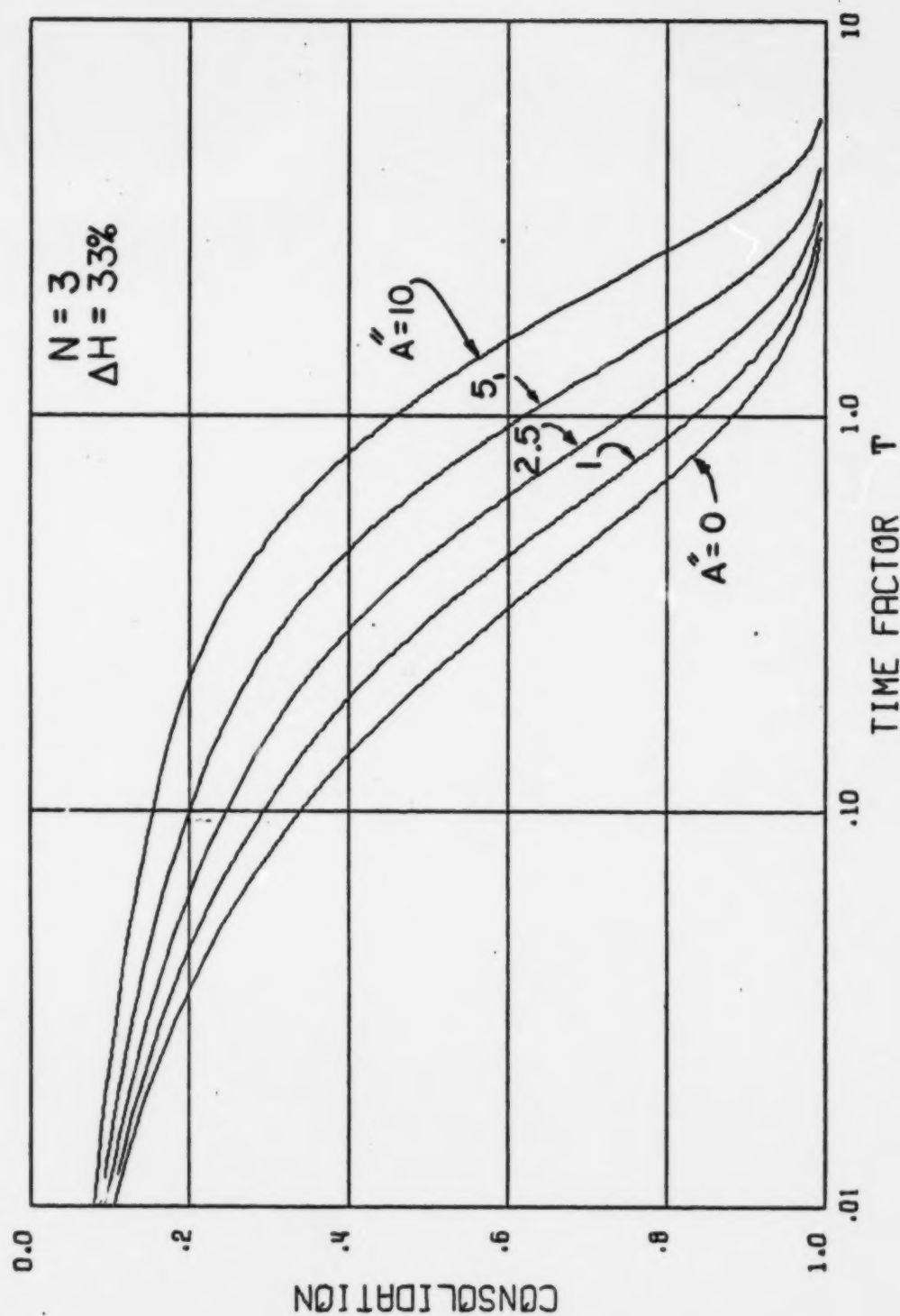


Figure 6-13: Degree of consolidation versus time for saturated material and variable A'' - thirty-three percent movement.

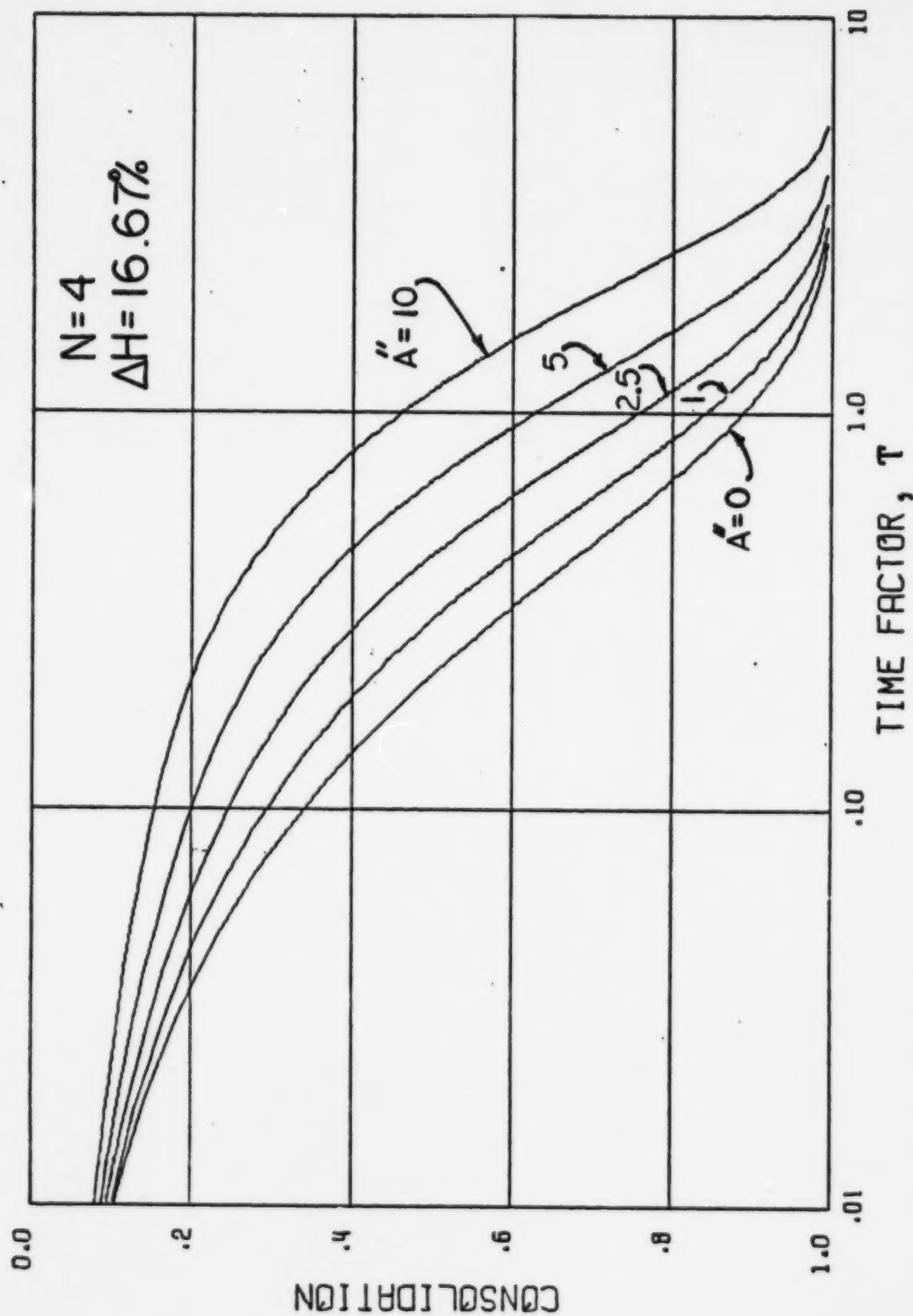


Figure 6-14: Degree of consolidation versus time for saturated material and variable A'' - sixteen and two-thirds percent movement.

$$\tau_s = \text{Log} \tanh \left(\frac{\phi_c \Delta p \lambda}{2} \right) + \tau_{sf} - \text{Log} \tanh \left(\frac{\phi_c \Delta p}{2} \right) \quad (6-14)$$

where τ_{sf} is the final structural time, that is, when the total creep is finished. Since the creep will continue for an unspecified time, equation 6-14 cannot be solved. Thus, equation 6-13 is solved using the finite difference method. Figure 6-15 shows the solutions obtained for $\phi_c \Delta p$ varying from one to 100. As the value of this term increases, the influence of the creep behavior upon the overall mechanical response decreases.

The creep function can be expressed in terms of the conventional time factor given in equation 6-9 using the ratio of the time factors. The ratio is

$$\tau_R = \frac{\tau}{\tau_s} = \frac{k_o(1 + e_o)}{\delta_w \phi_c H^2} \frac{1}{\theta} \quad (6-15)$$

This ratio may vary from near zero to more than one; the lower values for materials with long creep times, while the upper ones are for creep of short duration relative to the time of fluid dissipation. If equation 6-15 is substituted into 6-13, the creep function becomes

$$\frac{\partial \lambda}{\partial \tau} = \frac{1}{\Delta p \phi_c \tau_R} \sinh (\phi_c \Delta p (\lambda - u)) \quad (6-16)$$

The effect of the two constants $\Delta p \phi_c$ and τ_R should be made clear at this time. The time ratio is simply the ratio of the two time systems. That is, the time for the pore pressure dissipation phase to reach completion relative to that for the structural creep to reach its limit. The constant

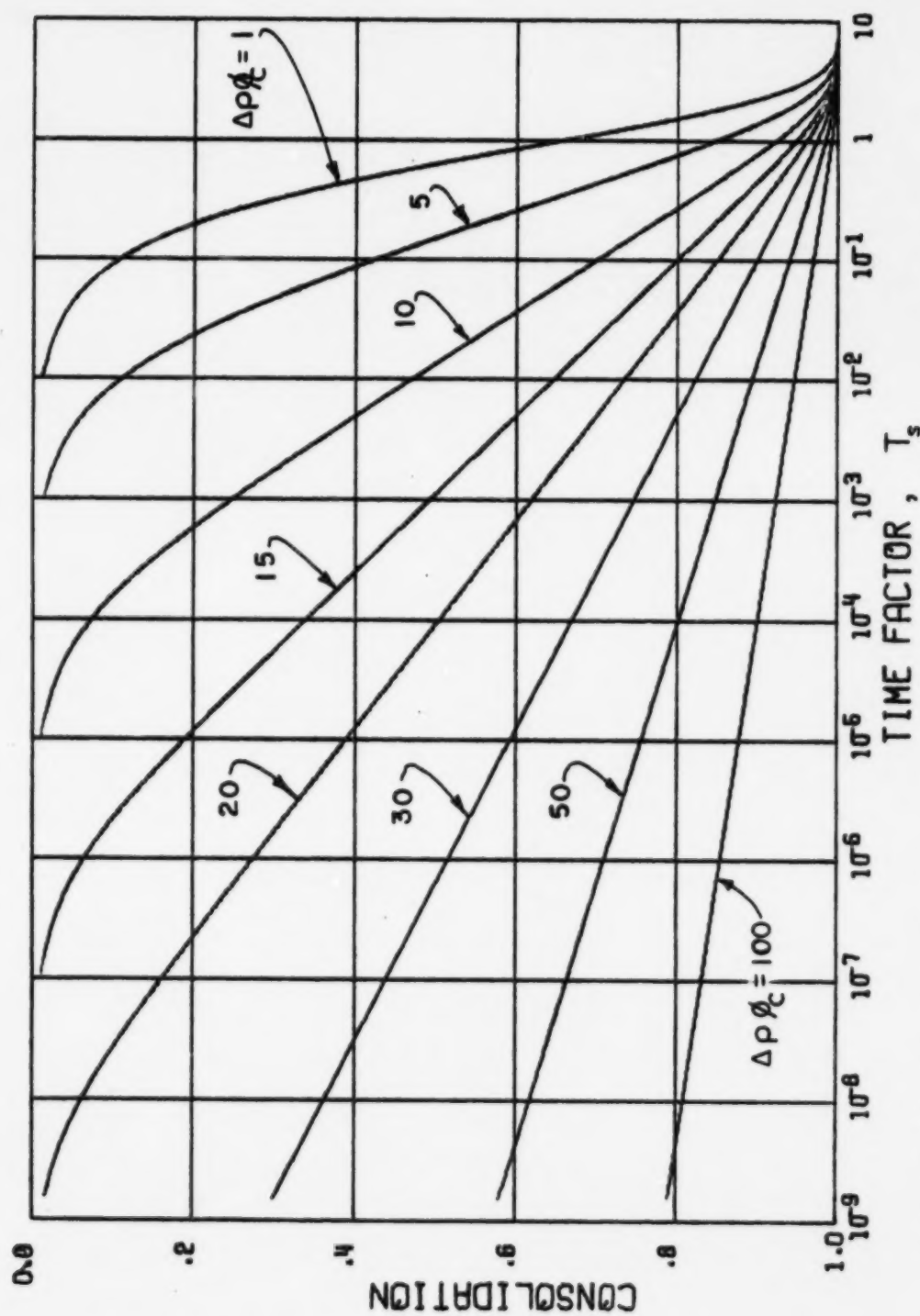


Figure 6-15: Creep versus structural time for case of no fluid pressure.

$\Delta p \phi_c$ is the intercept on the strain rate axis of the linear portion of the curve of the logarithm of strain rate versus principal stress difference. This constant gives the general magnitude of the structural creep in the material.

Summary

In this chapter the functions developed in the previous one are introduced into the equations developed in Chapter IV and using these new functions, the behavior of both mechanisms of consolidation are investigated. Initially, the fluid pressure dissipation equation is studied with only the classical fluid pressure term and the coefficient involving the void ratios. The response developed by the use of equation 6-11 is slower than that of the classical model. In addition, the greater the amount of volume change, more time required for completion of consolidation, and the time of consolidation also increases with increase in the power of the void ratio term.

The model's response is also greatly affected by the inclusion of the non-linear term in equation 6-12. As the value of the coefficient of the non-linear term (A'') increases, the speed of response decreases.

The creep function is solved for creep alone, and then put in terms of dimensionless time. The results show that for larger load increments, the effect of the pore pressure dissipation is greater relative to creep effects. This is in agreement with earlier statements and the data shown in Figure 4-4.

CHAPTER VII

COMBINED CREEP AND PRESSURE DISSIPATION FOR SATURATED CASE

Introduction

Although it is a single process, the consolidation of a material can be modeled by two distinct mechanisms, excess pore pressure dissipation and structural creep. In the previous chapter, deviations from the classical Terzaghi model of the behavior of the proposed pore fluid model for a saturated material are studied, as well as the behavior of the creep model. In this chapter, the two processes will be combined and the influence of the different variables in each will be analyzed. As stated at the end of Chapter IV, the procedures for the simultaneous solution of the two equations are given in Appendix A.

Behavior of Combined Model with Total Saturation

The mechanical model for the dual mechanism of settlement is obtained by solving equations 6-10 and 6-13 simultaneously using the time relationship expressed by equation 6-15. For the case of total saturation during the settlement process, the governing equations are

$$\frac{\partial \lambda}{\partial \tau} = \left(\frac{e}{e_0} \right)^N \left(\frac{1+e_0}{1+e} \right)^2 \left(\frac{\partial^2 u}{\partial z^2} + A'' \left(\frac{\partial u}{\partial z} \right)^2 \right) \quad (7-1)$$

and

$$\frac{\partial \lambda}{\partial \tau} = \frac{1}{\Delta p \phi_c \tau_n} \sinh(\Delta p \phi_c |\lambda - u|) \quad (7-2)$$

For these equations, the great variations in the behavior of the pressure dissipation model due to slight changes in its parameters are diminished considerably by the creep effects, and in some cases these parameters have little or no effect on the overall behavior.

Classical Case

The first case to be investigated is that of the classical term only in equation 6-10, or

$$\frac{\partial \lambda}{\partial T} = \frac{\partial^2 u}{\partial Z^2} \quad (7-3)$$

and equation 6-13. Figure 7-1 shows various plots for different values of $\Delta p \theta_c$ and T_R , as well as the no creep case, or classical Terzaghi solution. In the cases with the same time ratio values, consolidation is completed at nearly the same Time Factor value, while variations in the constant $\Delta p \theta_c$ cause differences in the shape and slope of the time-settlement curves. The response of the model with creep extends over a larger interval of Time Factor than that of the model without.

Figure 7-2 shows the mid-plane pore pressure versus Time Factor for the different cases shown in Figure 7-1. Although the initial pressure dissipation in the combined model is faster than that in the Terzaghi model, the pressures tend to remain for a considerably longer time in the model. For a given time ratio, the higher the value of $\Delta p \theta_c$, the closer the model's behavior to that of the classical model. Also, the lower the time ratio, the sooner the excess fluid pressures will dissipate.

A family of curves for a particular time ratio (100) and different

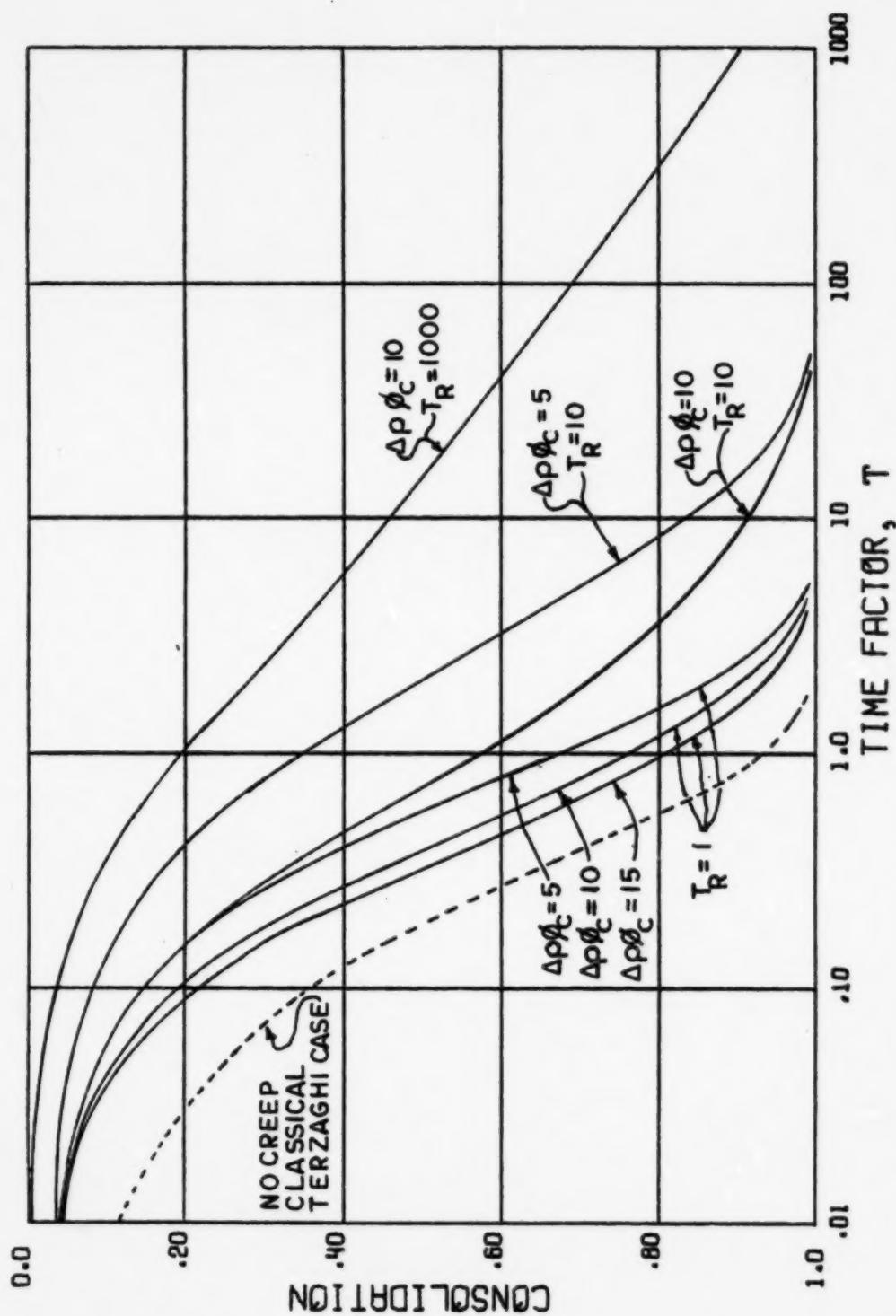


Figure 7-1: Time-settlement curves for combined pressure dissipation-creep model.

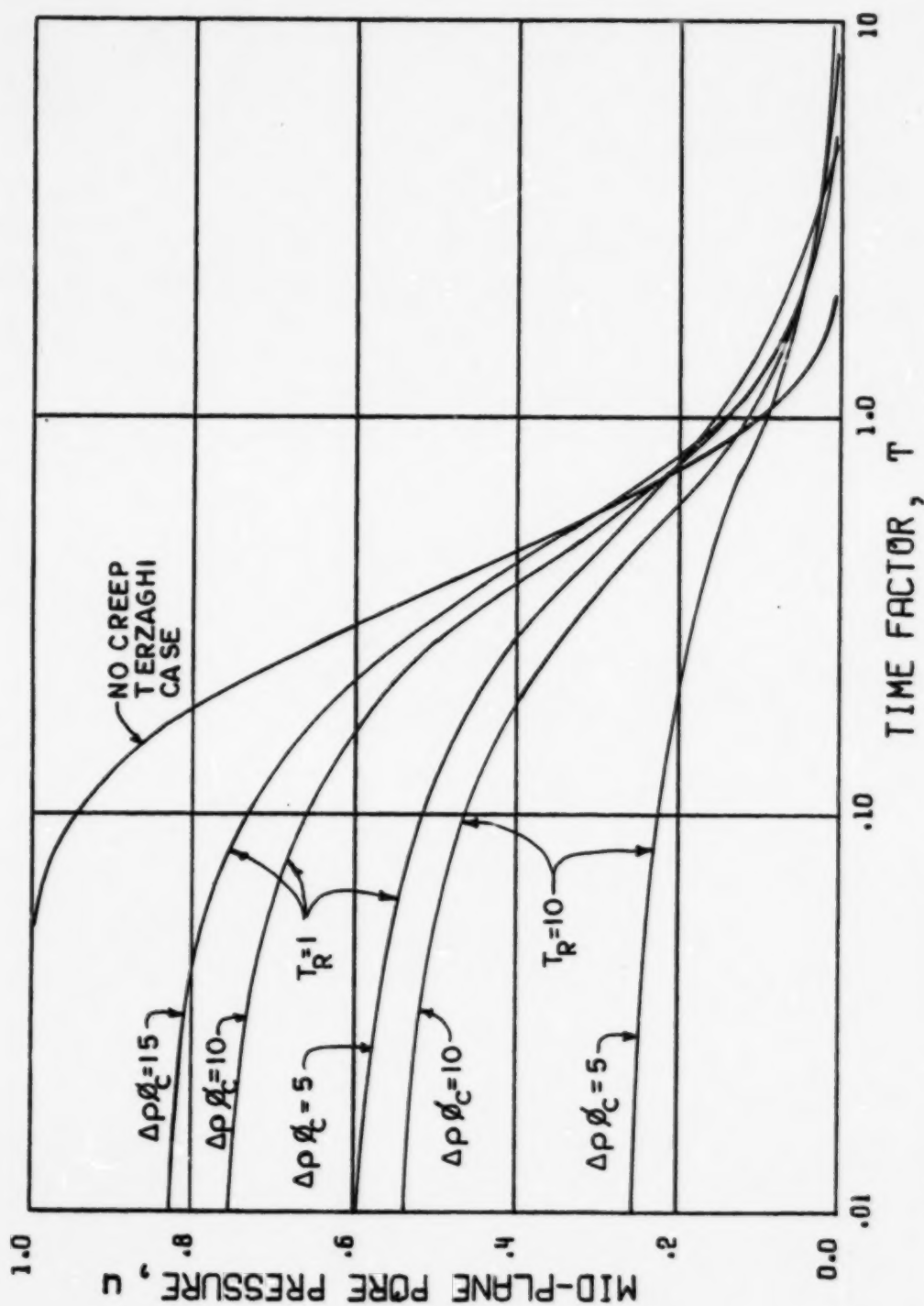


Figure 7-2: Mid-plane pore pressure versus time factor for combined model and Terzaghi model.

values of $\Delta p \emptyset_c$ is shown in Figure 7-3, along with the classical case. The consolidation process for the family of curves terminates at about the same Time Factor for all but the classical case, but the time-consolidation paths followed by the various cases differ considerably, depending upon the value of $\Delta p \emptyset_c$. For $\Delta p \emptyset_c = 5$, the pore pressure dissipation process is finished with nearly 80 percent of the settlement still to occur. For $\Delta p \emptyset_c = 30$, however, less than 20 percent of the total movement remains to be completed when the pressure dissipation is finished. Thus, the greater the value of $\Delta p \emptyset_c$, the greater the portion of the settlement that will occur during the pressure dissipation phase.

Finite Strain Case

The second case considers A'' equal to zero, or

$$\frac{\partial \lambda}{\partial T} = \left(\frac{e}{e_0} \right)^N \left(\frac{1+e}{1+e_0} \right)^2 \left(\frac{\partial^2 u}{\partial z^2} \right) \quad (7-4)$$

and equation 7-2. The relative importance of the value of the time ratio begins to appear in this case. Figures 7-4 and 7-5 show the behavior of materials with a time ratio of one, an N value of three and $\Delta p \emptyset_c$ equal to five and ten, and volume change, or settlement, values of 10 and 40 percent. Figure 7-4 shows that curves with the higher $\Delta p \emptyset_c$ values are ahead of the curves for lower $\Delta p \emptyset_c$ values in the amount of settlement at any given time. Figure 7-5 shows that the pressure dissipation phase for the lower $\Delta p \emptyset_c$ values initially proceeds faster than for the higher values, but the curves nearly overlap after a dimensionless time value of about unity. However, for the lower $\Delta p \emptyset_c$ values, the excess pressure

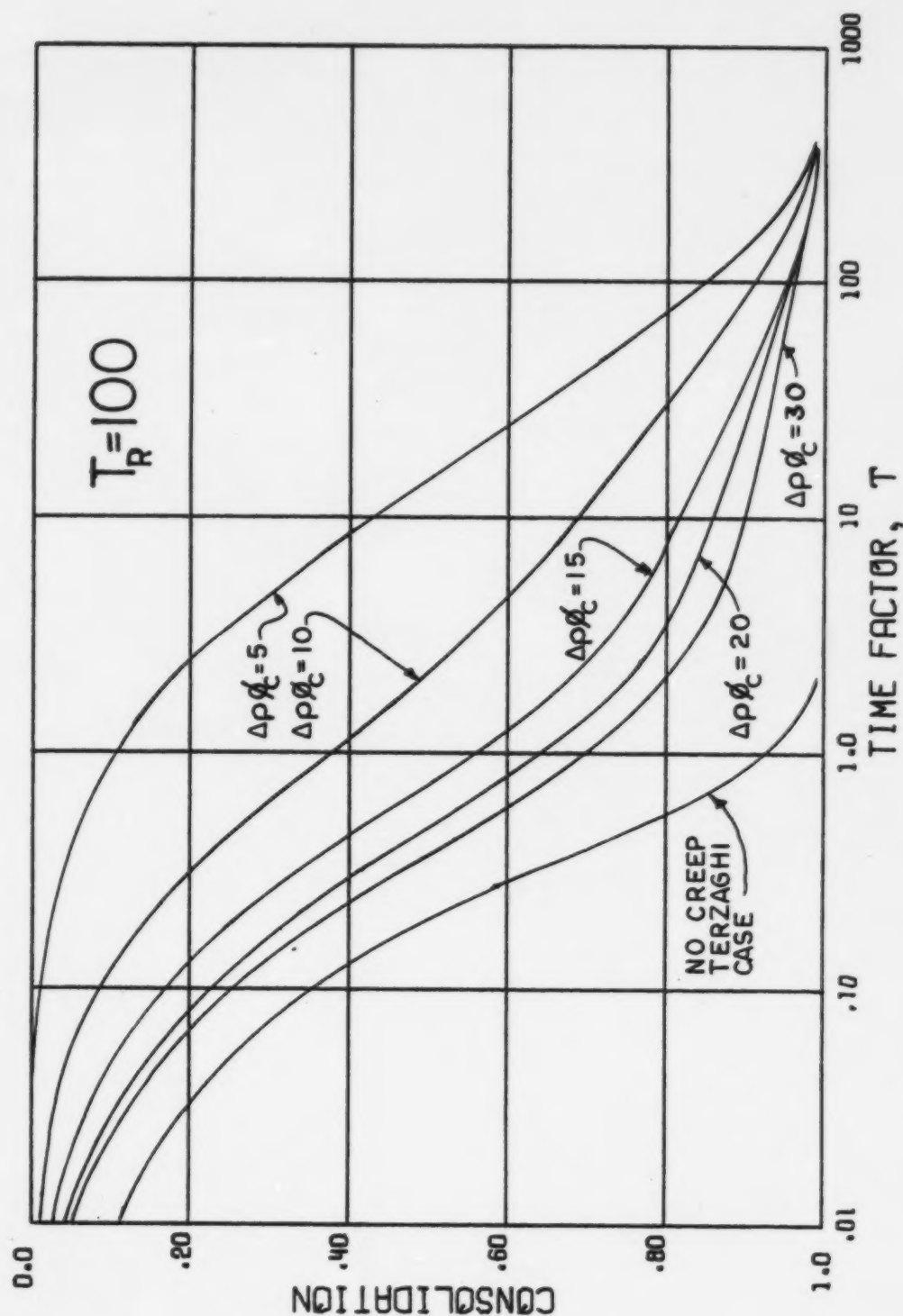


Figure 7-3: Family of curves for $T_R=100$ and the classical solution.

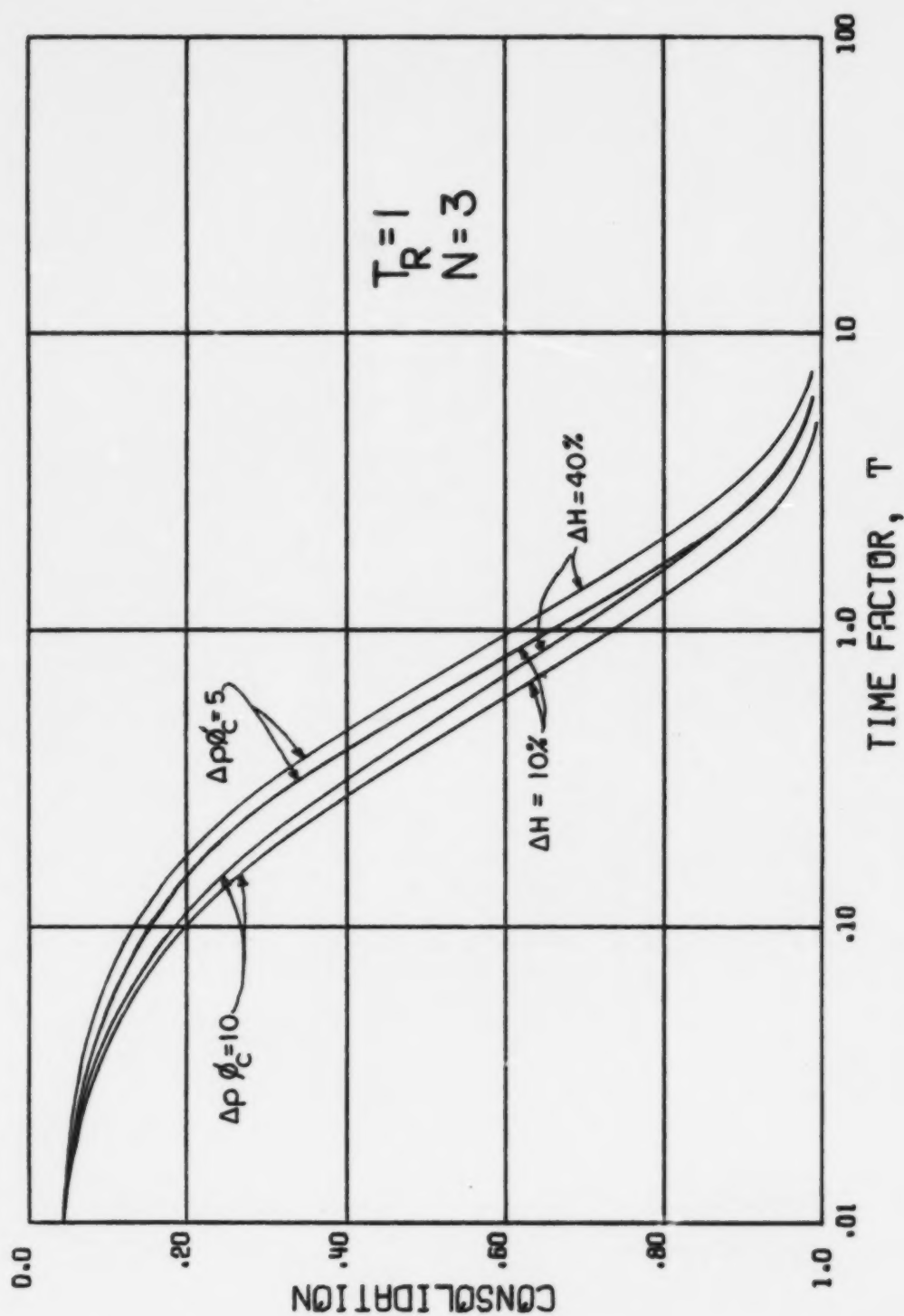


Figure 7-4: Time-settlement curves for two $\Delta p / \sigma_c$ values.

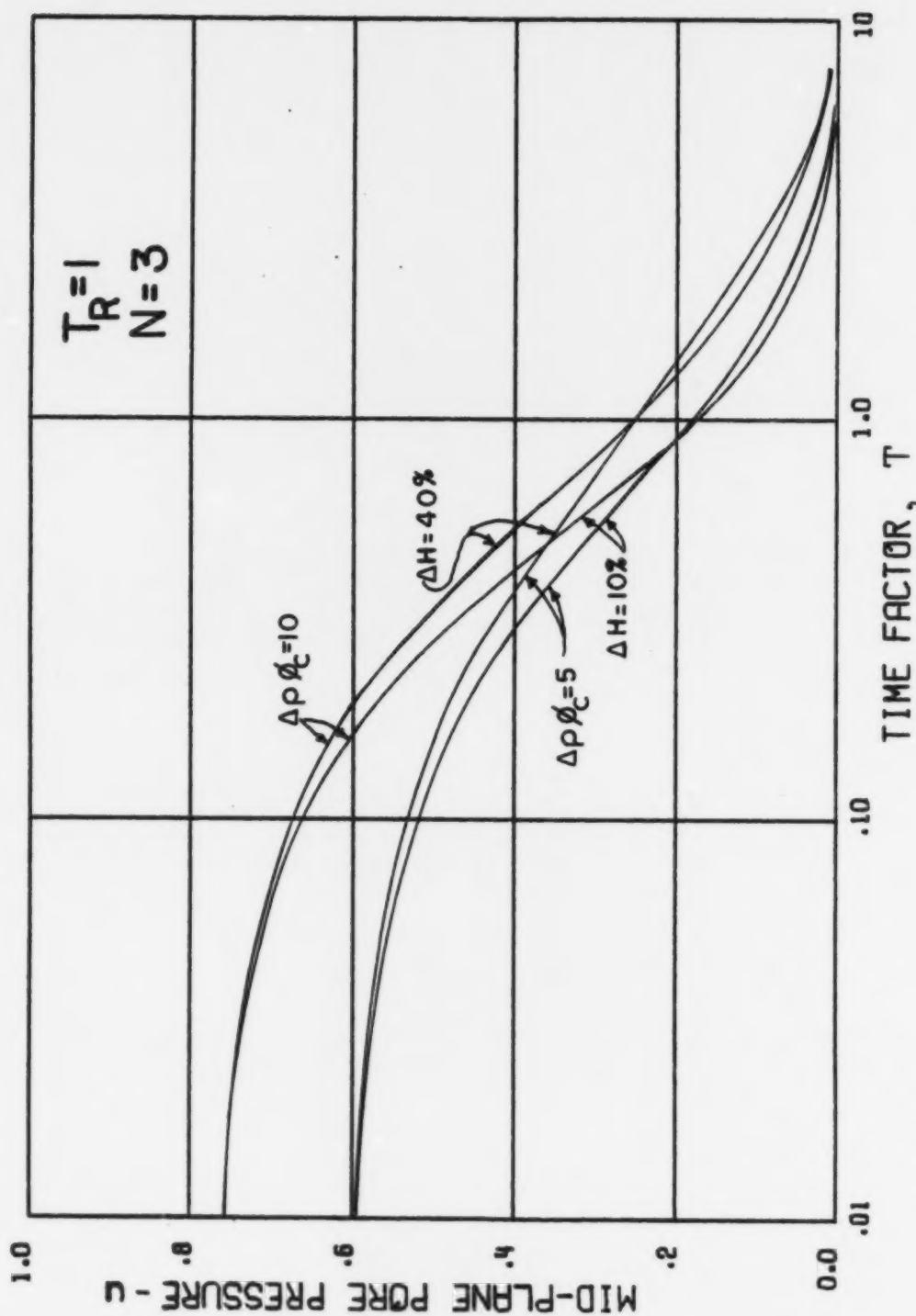


Figure 7-5: Mid-plane pore pressure-time curves for cases shown in figure 7-4.

dissipation tends to continue for a longer time. It should be noted that the effect of variation in $\Delta p \emptyset_c$ on the pore pressure-time relationship for a particular percentage settlement value is the opposite of that on the consolidation-time relationship.

Figures 7-6 and 7-7 show the same type of curves with varying N values, and 20 percent ultimate settlement. These curves show a slight delay in the settlement-time and pressure dissipation response of the model with increasing values of N .

The overpowering influence of the creep behavior is shown in Figures 7-8 and 7-9. The first of these figures shows the behavior of four cases for different percentages of settlement, and except for a slight variation around a dimensionless time of unity, the four curves are virtually identical. In the second graph, the value of N varies from two to five and there are almost no variations in the curves. For both groups, the pore pressure-time graphs of four different cases produce a single curve, but the time required for total dissipation of excess fluid pressure differs. For the group shown in Figure 7-8, the time variance is negligible although observable. However, the dissipation time corresponding to the various N values for the cases in Figure 7-9 varies from 10.12 to 14.34, for a time difference of 40 percent. This effect, however, is not visible in Figure 7-9 because of the overshadowing effect of settlement due to creep.

The general model for the saturated case will consist of equations 7-1 and 7-2. Figure 7-10 shows curves for $\Delta p \emptyset_c$ equal to ten, a time ratio of one, an N value of three, settlement values of 20 percent, and A'' values of 0.0, and 10.0. The variation of the time-consolidation curves for the different A'' values does not approach that in Figures 6-13.

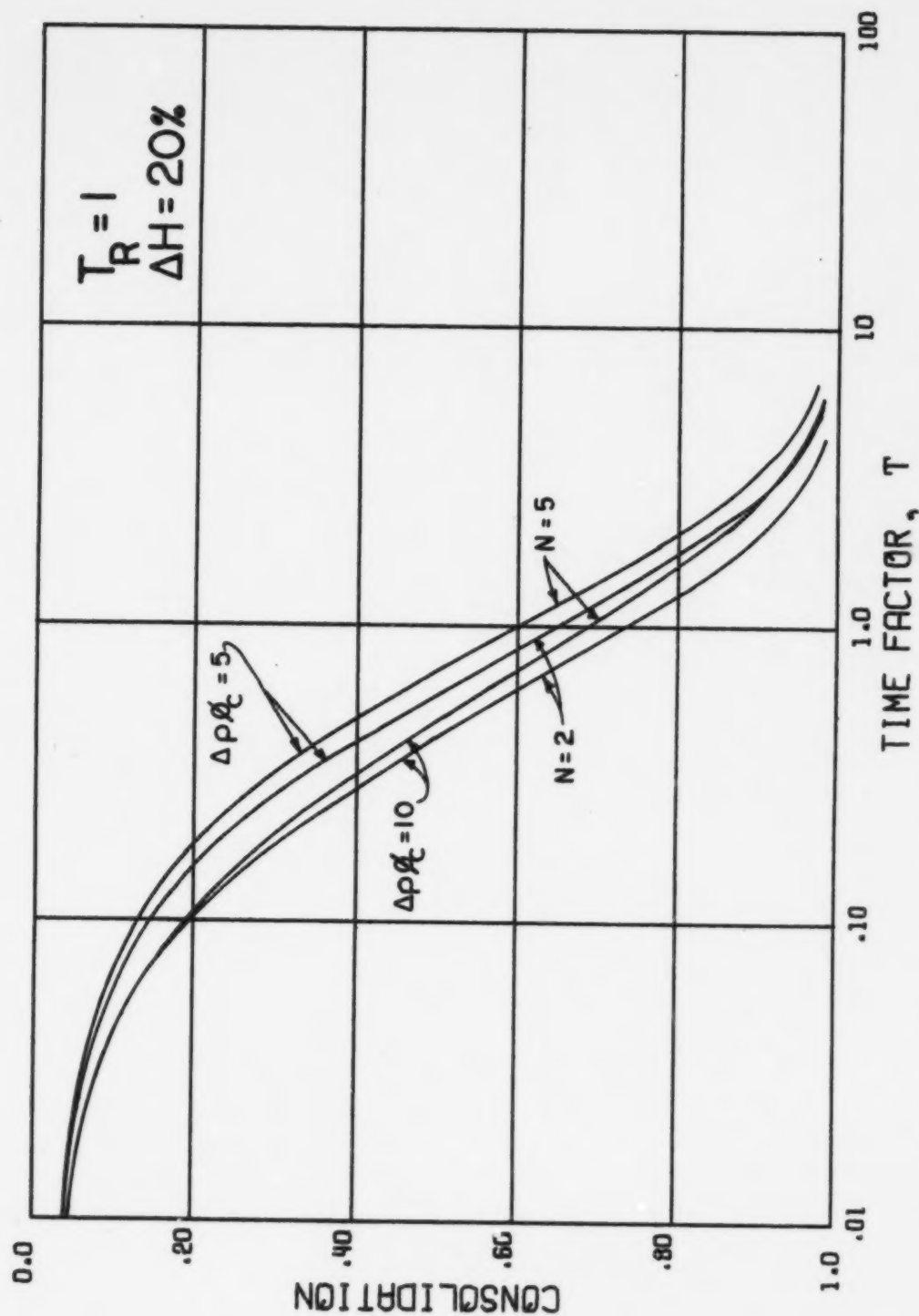


Figure 7-6: Time-settlement curves for twenty percent movement and $\Delta p\phi_c$ equal to five and ten.

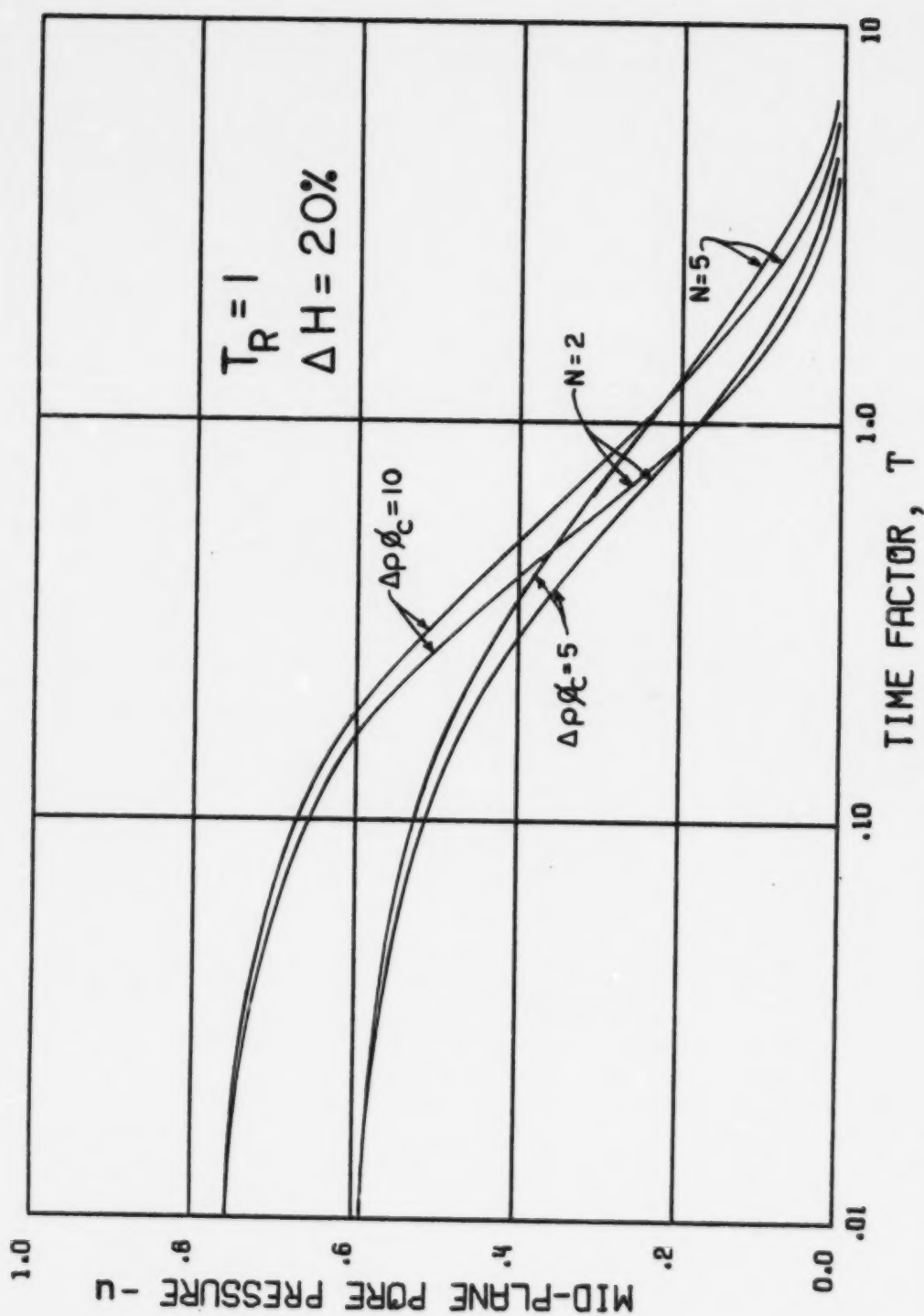


Figure 7-7: Mid-plane core pressure-time curve corresponding to cases shown in figure 7-6.

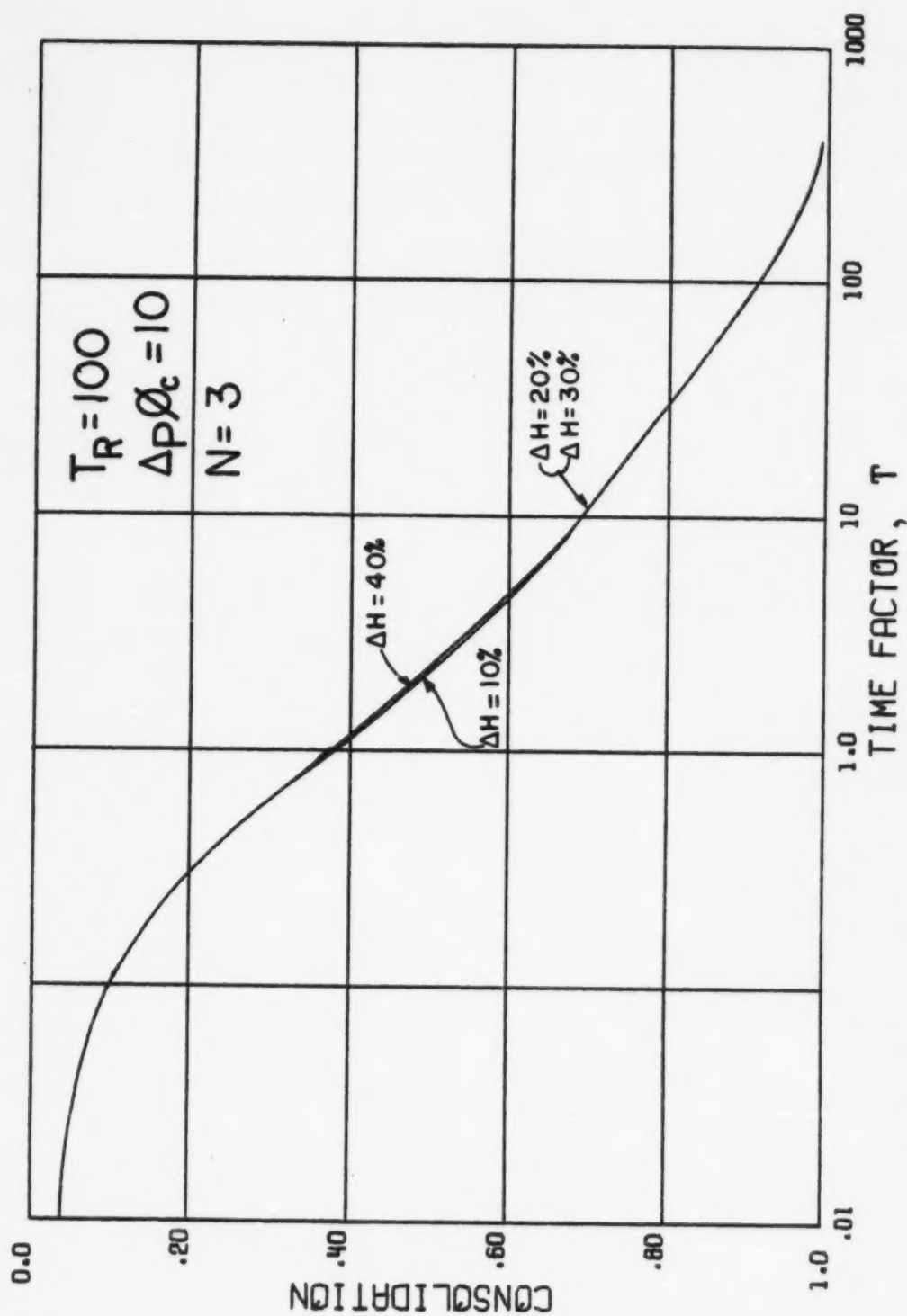


Figure 7-8: Time-settlement curves for high time ratio values, and varying amounts of volume change.

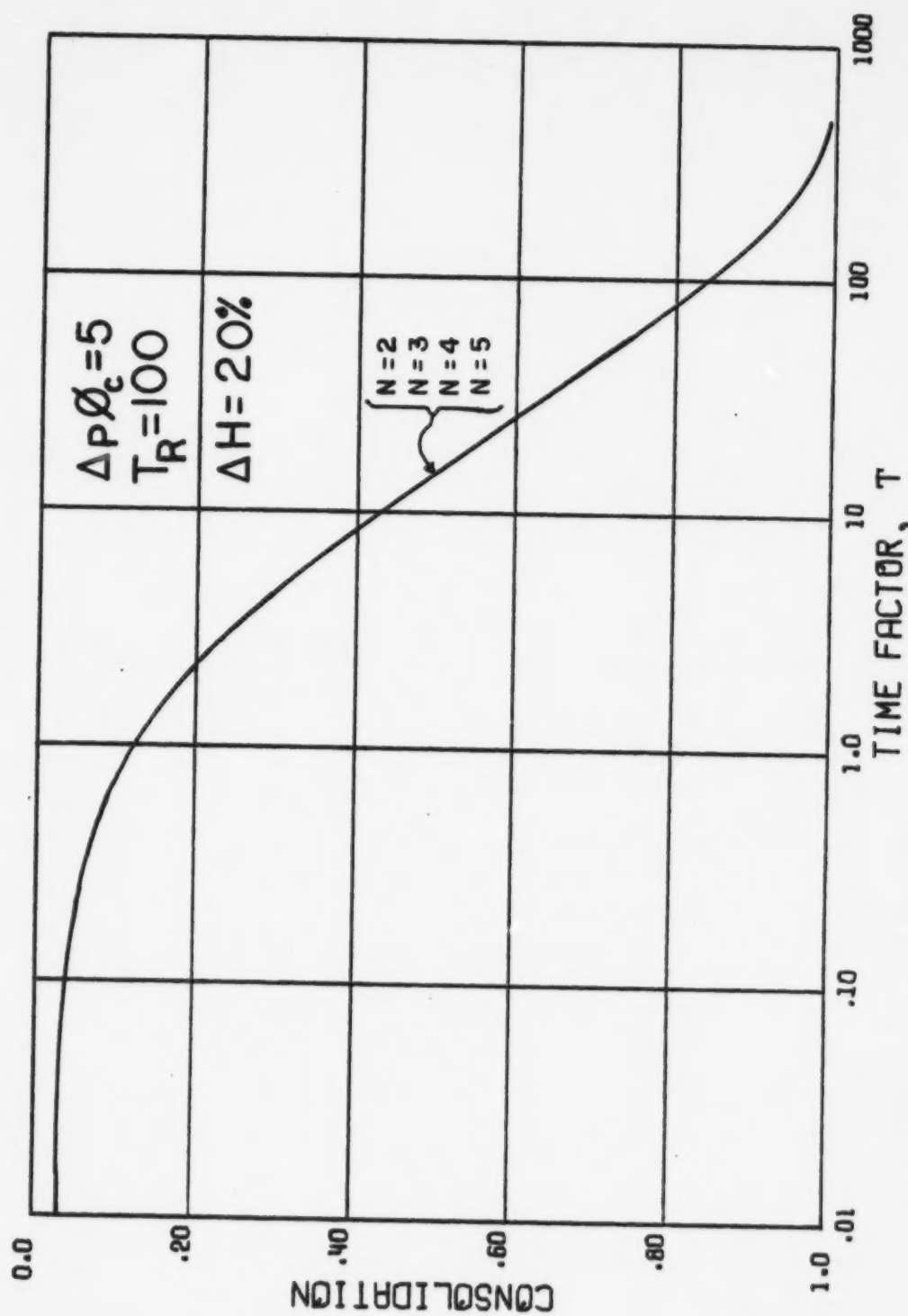


Figure 7-9: Time-settlement curves for high time ratio and varying N values.

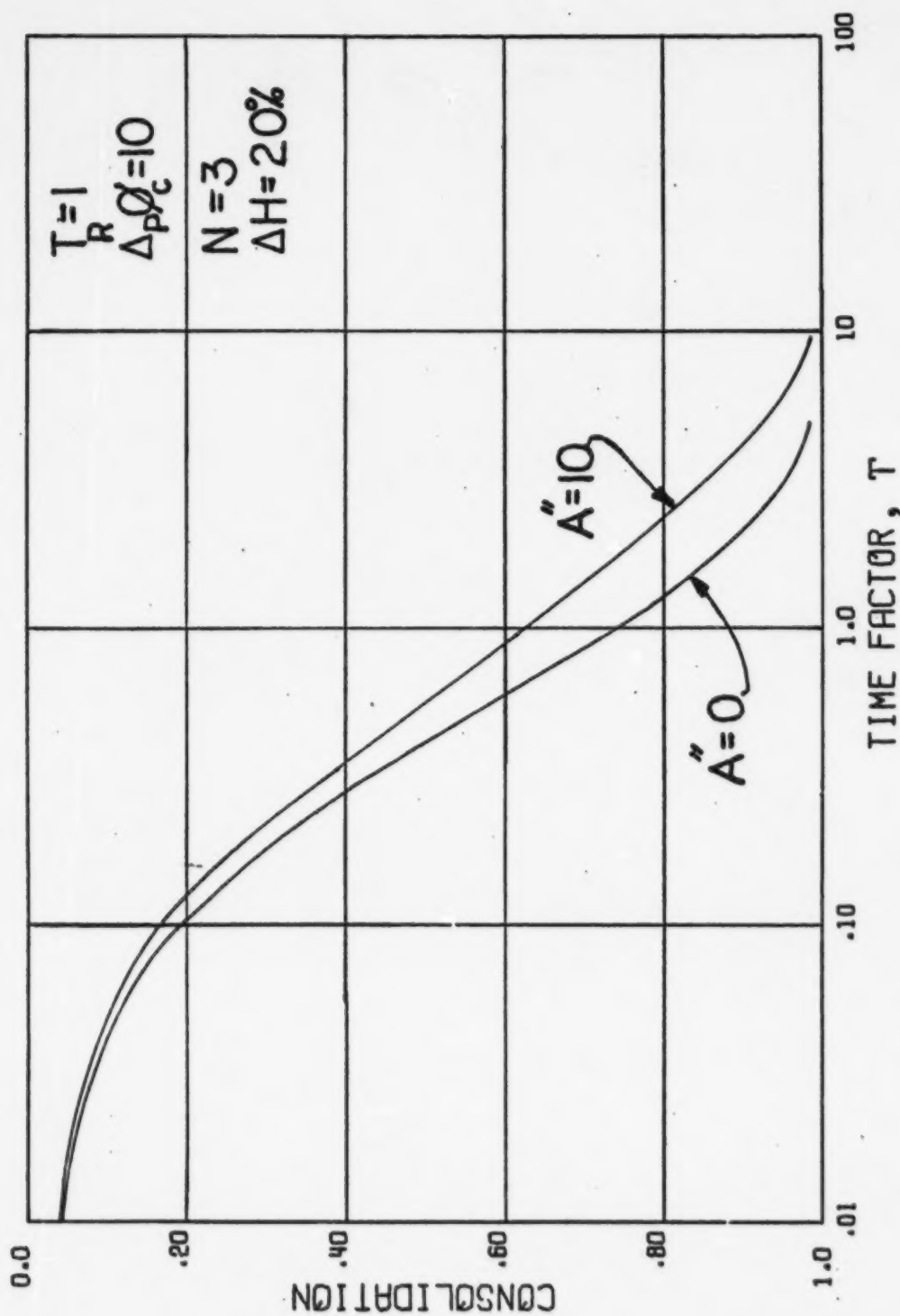


Figure 7-10: Time-settlement curves for varying values of A'' .

and 6-14, again indicating the influence of the creep. The dominance of creep is shown again in Figure 7-11 where, for $\Delta p \emptyset_c$ equal to ten, time ratio of 100, and A'' varying from 0.0 to 10.0, as in Figures 7-8 and 7-9, the four cases are indistinguishable. Figure 7-12 shows the pore pressure behavior for the same cases shown in Figure 7-11. Again, the four cases are indistinguishable, and the dissipation of pore pressure is finished nearly at the same Time Factor.

Summary

The variations in the pore pressure dissipation equation are not nearly as important when the creep relation is incorporated in the behavior as they are without the creep. The graphs of Chapter VI show great variation due to slight changes in the parameters, while the graphs of Chapter VII show little or no variation. Figure 7-13 shows solutions of equations 7-1 and 7-2 with values of parameters that will give large variations in the time-settlement behavior. The lower curve is for a material with $\Delta p \emptyset_c$ equal to ten, a time ratio of one, N equal to two, A'' equal to 0.0 and 10 percent settlement. The upper curve has the same $\Delta p \emptyset_c$ and time ratio, N is equal to five, A'' is equal to 10.0 and settlement is 40 percent. Overall, the variation for the two cases is not nearly as great as those in Chapter VI although the dimensionless time to completion in the longer case is about twice that in the faster case.

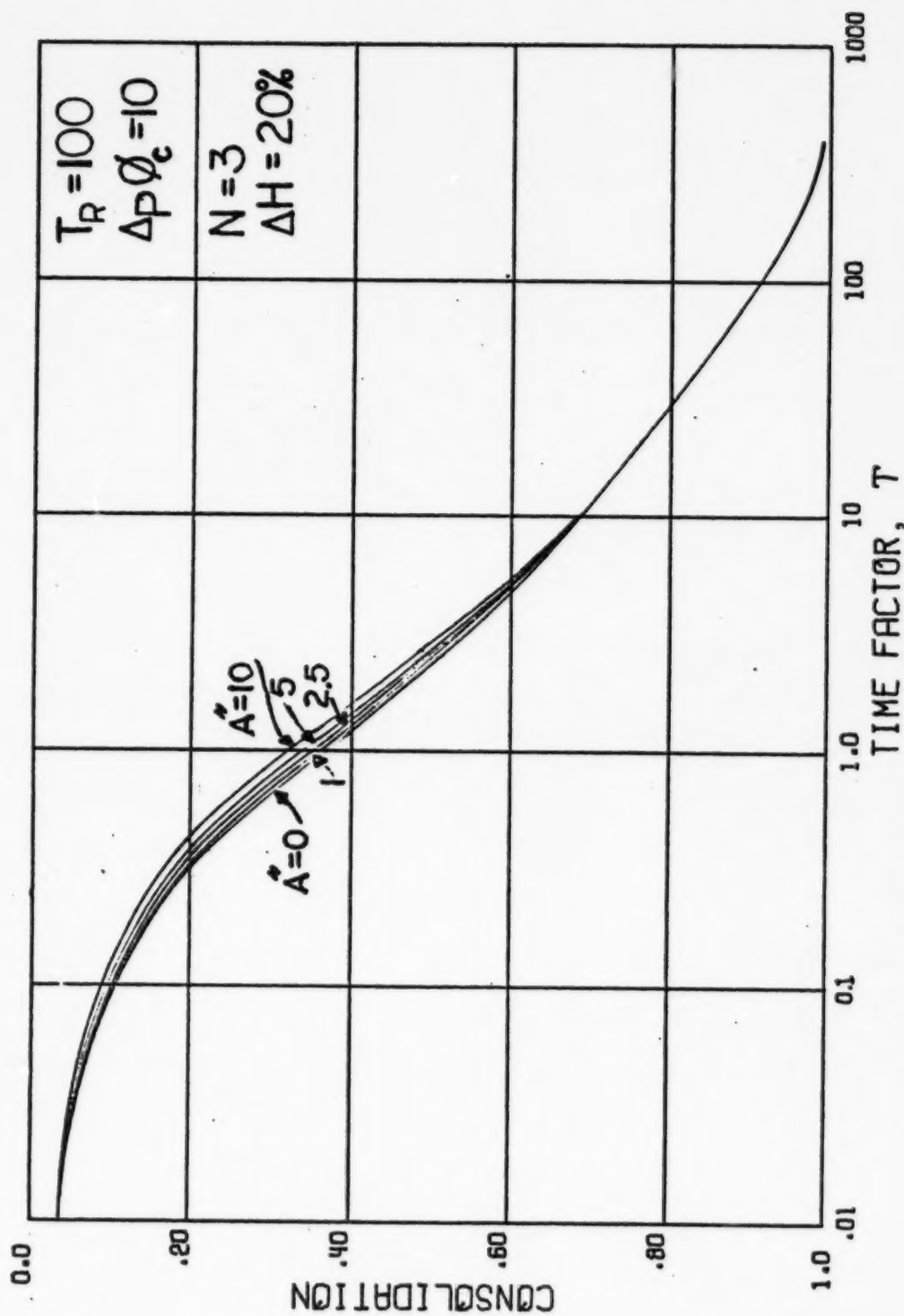


Figure 7-11: Time-settlement curves for high time ratio, showing the dominance of creep.

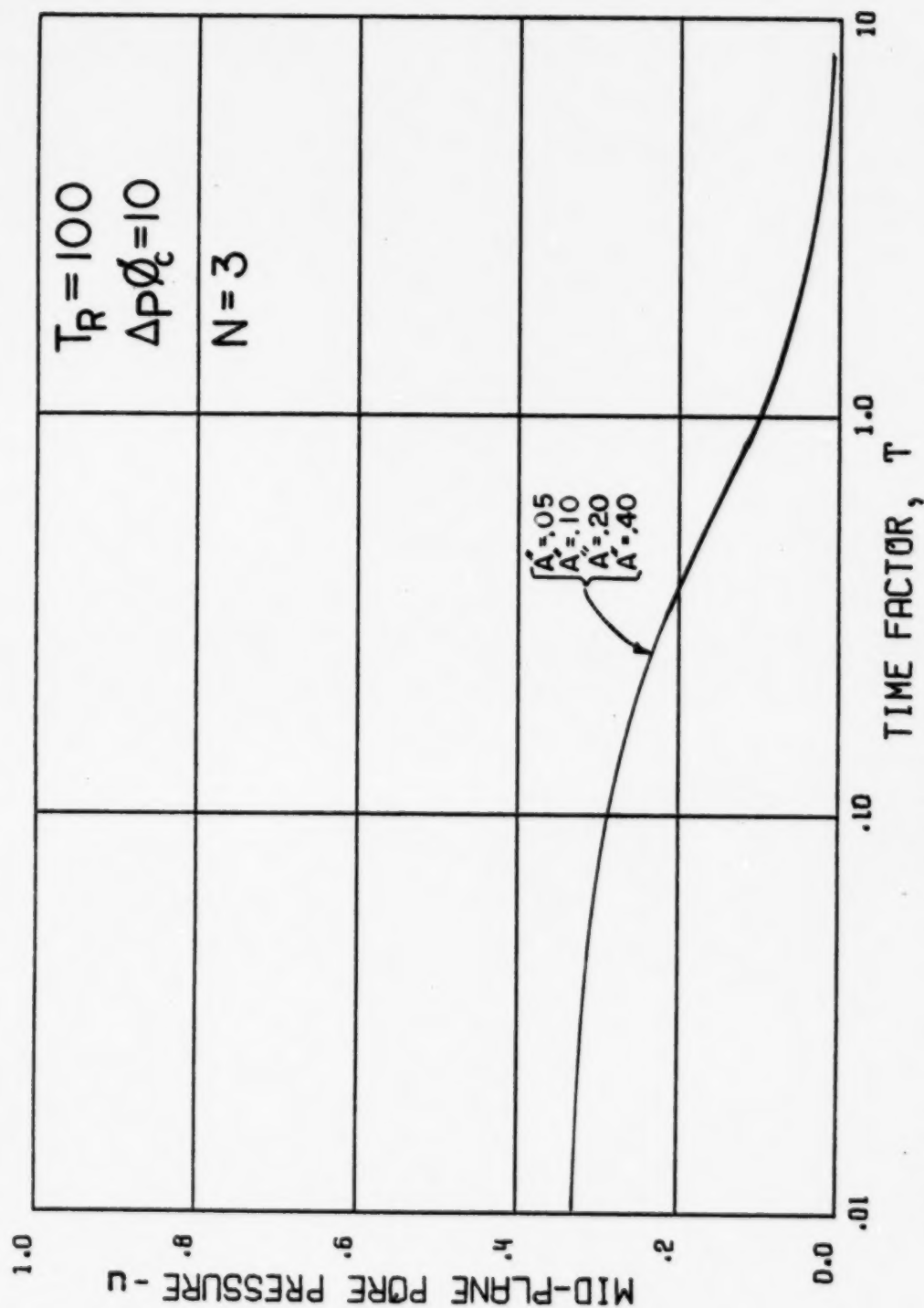


Figure 7-12: Time - mid-plane pore pressure curves for cases shown in figure 7-10.

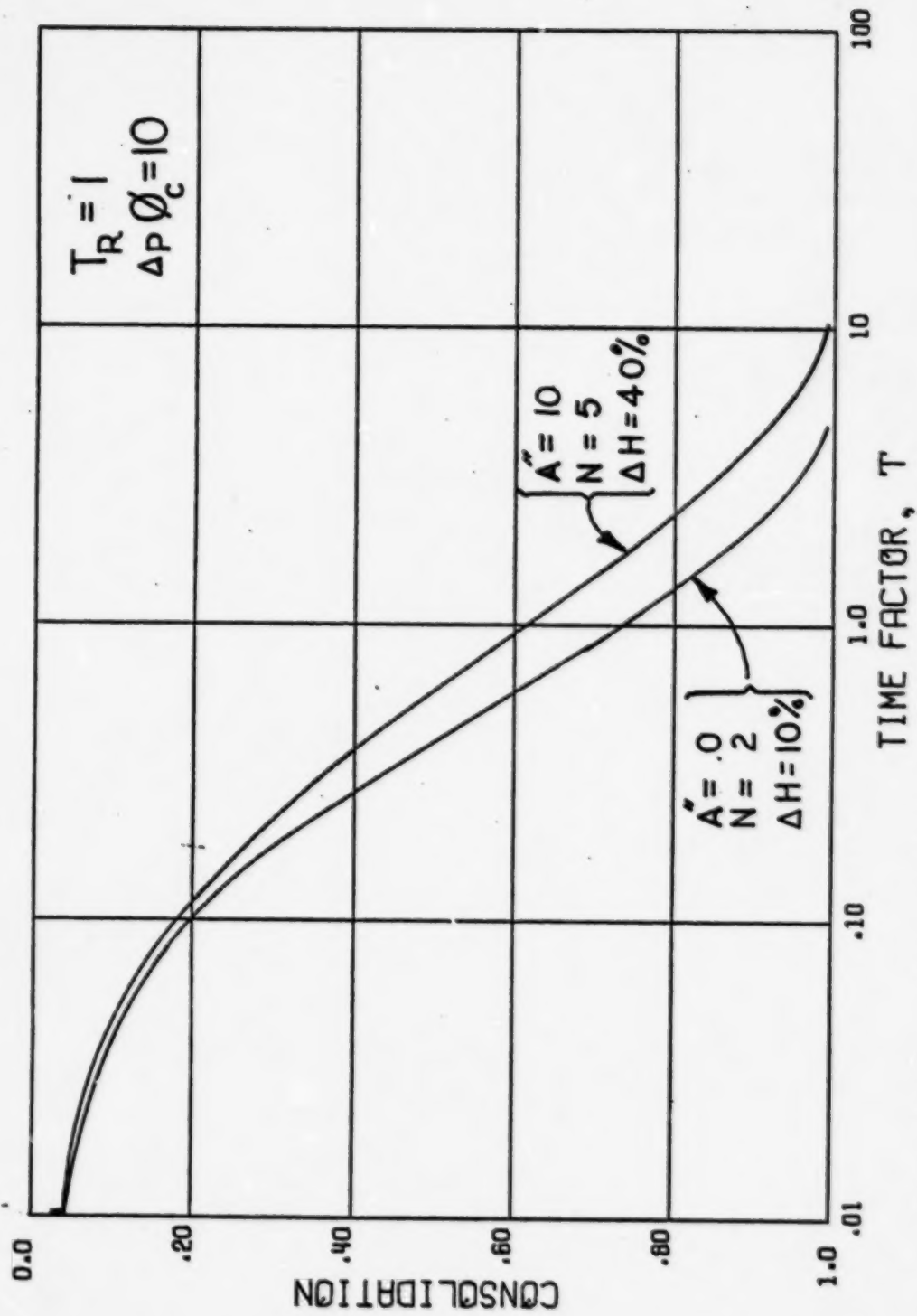


Figure 7-13: Time-settlement curves with different parameters.

CHAPTER VIII

UNSATURATED BEHAVIOR

Introduction

The analysis of unsaturated consolidation of a material is quite complicated because of the complexity of pore fluid behavior. The first case of interest is the comparatively simple one of partial saturation with inert solids and pore fluids. The second includes the effect of changes in the material behavior due to biological and/or chemical breakdown. The range of the degree of saturation during consolidation is important because the fluid characteristics change and the mechanism of consolidation changes with saturation. At high degrees of saturation, a decrease in the water content will produce a decrease in the transmissibility and thus a longer period of fluid flow or a longer consolidation time. Below the residual saturation, however, the pore fluid movement will be primarily that of gas; the fluid flow will be very rapid, and in fact will have negligible influence upon the rate of consolidation, which will be determined by the creep or structural action. Unfortunately, no information is available on the influence of saturation on the creep behavior of organic materials, and it will be assumed that the viscous characteristics do not change with saturation.

Multi-phase Pore Fluid

As mentioned in the earlier chapters, when gas is part of the pore fluid, its compressibility and the effect it has on the stress distribution make the mechanical behavior much more difficult to describe. When the load is applied, the unsaturated material initially undergoes an instantaneous

movement due to the compression of the gas; the magnitude of this compression depends on the degree of saturation and the stiffness of the solid matrix. In addition, as the excess pressure in the fluid is reduced with time, the confining pressures on the gas are also reduced and the gas will expand, changing the degree of saturation as consolidation progresses. If the load is assumed to be applied instantaneously, the resulting instantaneous compression of the system can be calculated using equations 5-44 and 5-45. First the fluid pressure in the system after loading is found, using equation 5-45, then the percent of the total height that is lost due to the compression is calculated with equation 5-44. With these results and equation 5-46, the initial conditions for the consolidation process are then known. The general governing equations are then 6-10 and 6-16. Equation 6-10 contains a term for the change in saturation with time which can be approximated by use of equation 5-49 and the assumption that the weight ratio of gas to liquid in the system is constant within each time increment (although it may be allowed to change instantaneously at the beginning of a time increment).

Unsaturated Inert Behavior of the Pore Fluid

If there is no biological or chemical activity in the material, then the pore pressure dissipation equation is

$$\frac{\partial x}{\partial T} = S_e^u \frac{U_g}{\Delta \rho} \left(\frac{e}{e_0} \right)^N \left(\frac{1+e_0}{1+e} \right)^2 \left(\frac{\partial u}{\partial z^2} + A'' \left(\frac{\partial u}{\partial z} \right)^2 \right) - \frac{e}{S} \frac{\partial S}{\partial T} \quad (8-1)$$

which is obtained from equation 6-10. The effective saturation, as defined in equation 5-12, varies from unity for a totally saturated material to

zero for a material at the residual saturation. This wide variation is magnified by the term being raised to a power which is possibly as large as five, and thus the coefficient will greatly influence the model's behavior.

The first case to be considered is that of unsaturated consolidation with no variation in the saturation value. The effective saturation will then be constant and the equation is

$$\frac{\partial \lambda}{\partial \tau} = s_e^{N_s} \frac{u_o}{\Delta \rho} \left(\frac{e}{e_o} \right)^N \left(\frac{1+e_o}{1+e} \right)^2 \left(\frac{\partial^2 u}{\partial z^2} + A'' \left(\frac{\partial u}{\partial z} \right)^2 \right) \quad (8-2)$$

To allow comparison of the results of this chapter and Chapter VI, the initial and boundary conditions will be taken to be those after the instantaneous compression has occurred.

The behavior of a material with N equal to three, A'' equal to zero, undergoing $16\frac{1}{2}$ percent total settlement, effective saturation equal to 90 percent, and N_s values varying from one to five is shown in Figure 8-1. These curves correspond with Figure 6-4, and the curve for the solution of equation 6-11 with N equal to three is also shown in Figure 8-1. The curve from Chapter VI falls under those of equation 8-2, meaning that the settlement is delayed by partial saturation. This delay increases with the increase of the power to which the effective saturation is raised. This should be expected since this will reduce the coefficient of the pore pressure terms, and reflects a lower fluid transmissibility. Also, it should be noted that both the pore fluid and consolidation graphs are clearly separated even at a time value of .01 for the solution of equation 8-2, while those of Chapter VI nearly coincide at this time value. The same case, except that the effective saturation is 70 percent, is shown in Figure 8-2.

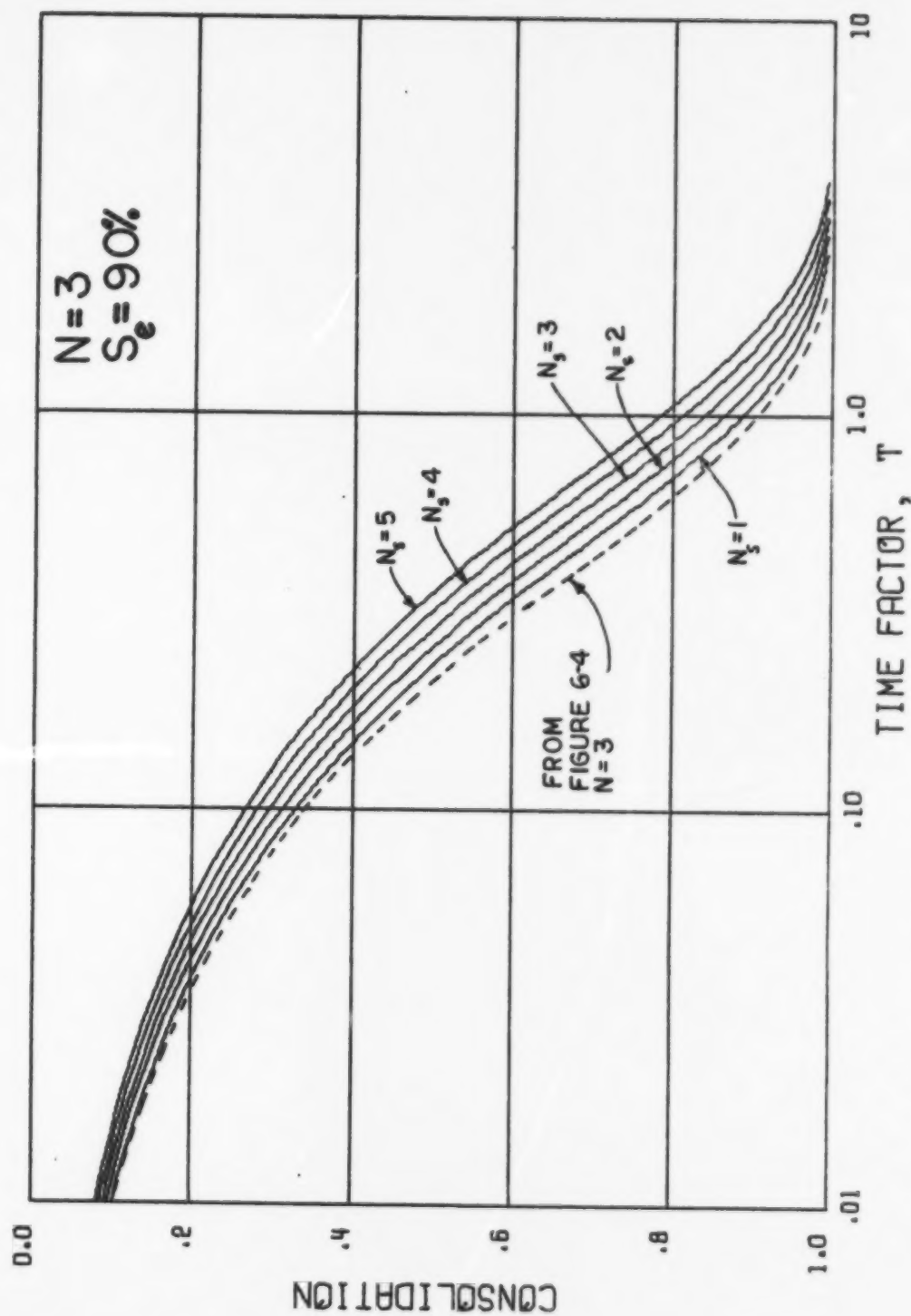


Figure 8-1: Time-consolidation curves for the effect of variations in N_s upon material behavior

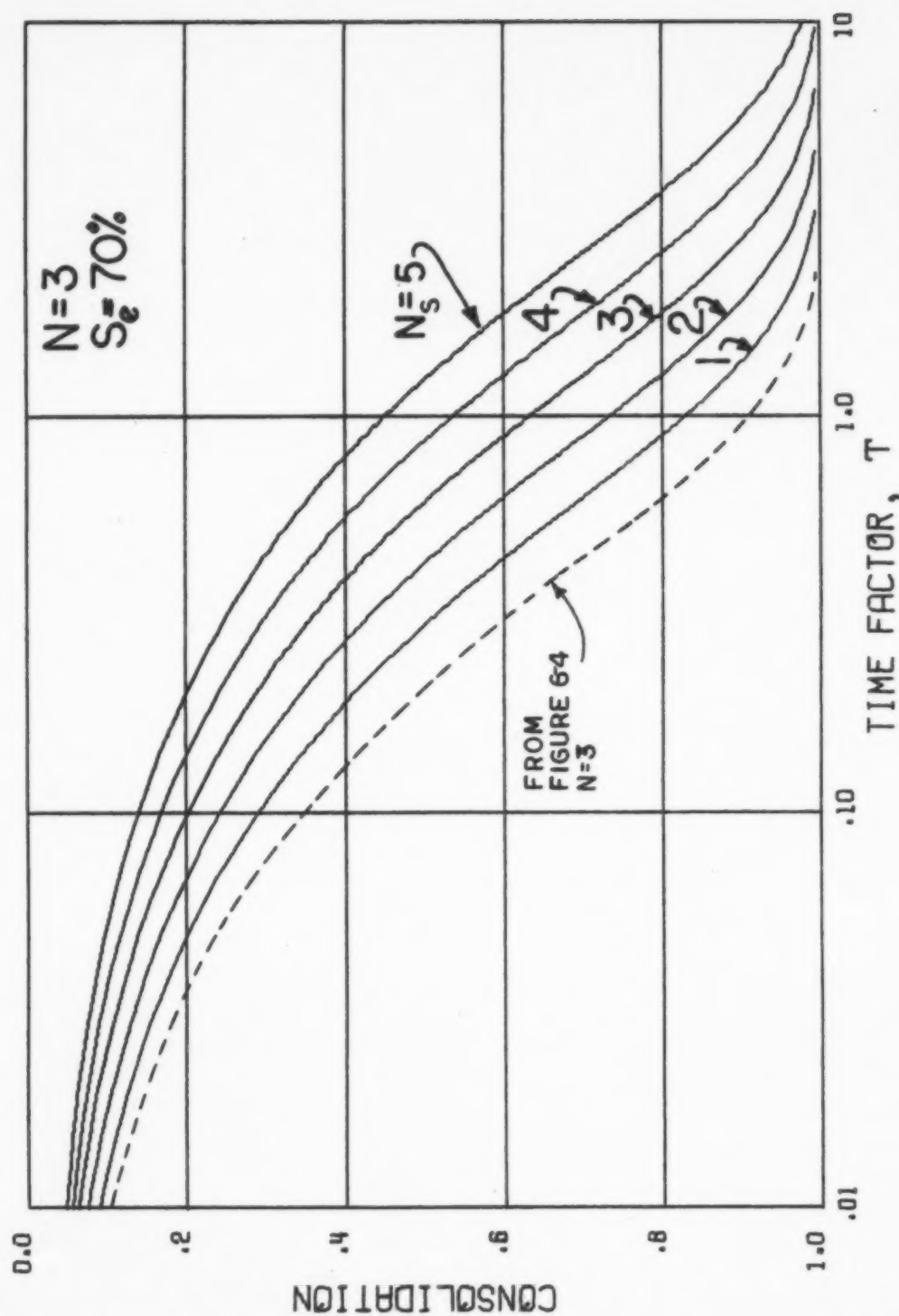


Figure 8-2: Time-consolidation curves for the effect of variations in N_s in a material with an effective saturation of seventy percent.

The solution from Chapter VI is also shown. Comparing the two figures clearly shows that the time of consolidation increases with a decrease in the effective saturation. The variable N_s is discussed by Brutsaert (1968), who says that N_s "would seem to depend mainly on the distribution of the pore sizes...for a medium with very uniform pore sizes, $n(N_s)$ may be as small as 2, whereas for a medium with a wide distribution of pore sizes, it may exceed 5...there is no agreement on the exact nature of $n(N_s)$. However, this may not be very important since recently (Ibrahim and Brutsaert, 1968) it has also been shown that, for example, for the problem of infiltration into a loam soil it makes very little difference whether $n = 3.0$, 3.5 or 4.5."

The effect of the effective saturation is shown more clearly in Figure 8-3, where the N value is three, N_s is one, the effective saturation varies from one to .10, and other variables are the same as those for Figures 8-1 and 8-2. The curve for total saturation ($S_e = 1$) is identical to the similar case in Chapter VI (Figure 6-4, $N = 3$) as it should be. The figure shows, as indicated before by Figures 8-1 and 8-2, that the rate of consolidation decreases with the effective saturation.

The effect of variations in A'' are shown in Figure 8-4, where the material has N equal to three, 16½ percent settlement, and 90 percent effective saturation. As in Figures 6-13 and 6-14, the lower the value of A'' , the faster the consolidation will occur.

Variable Saturation During Consolidation

Unsaturated consolidation will not proceed without changes in the degree of saturation because of the decrease in the confining pressures upon the

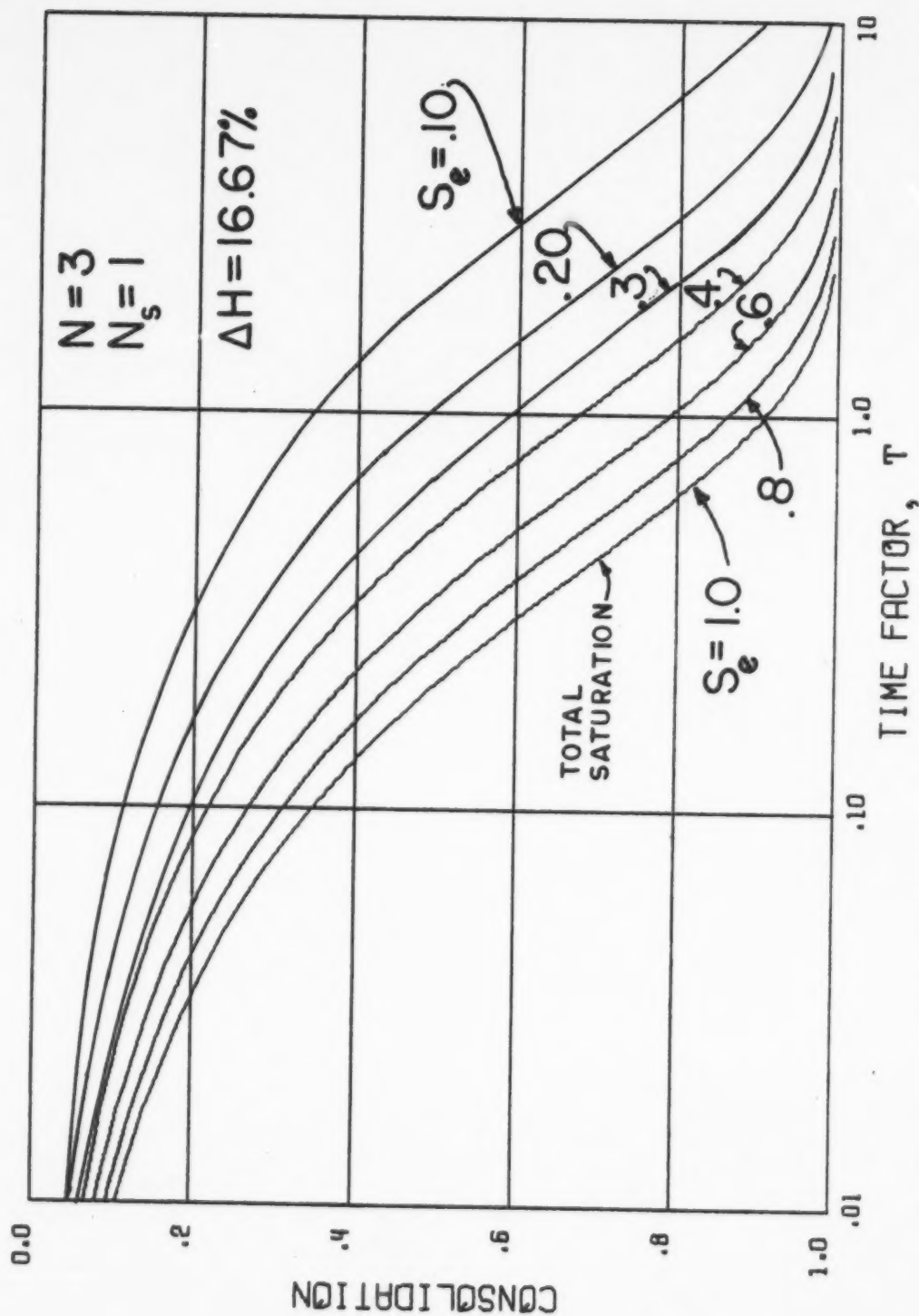


Figure 8-3: Time-settlement curves showing the effect of changes in effective saturation.

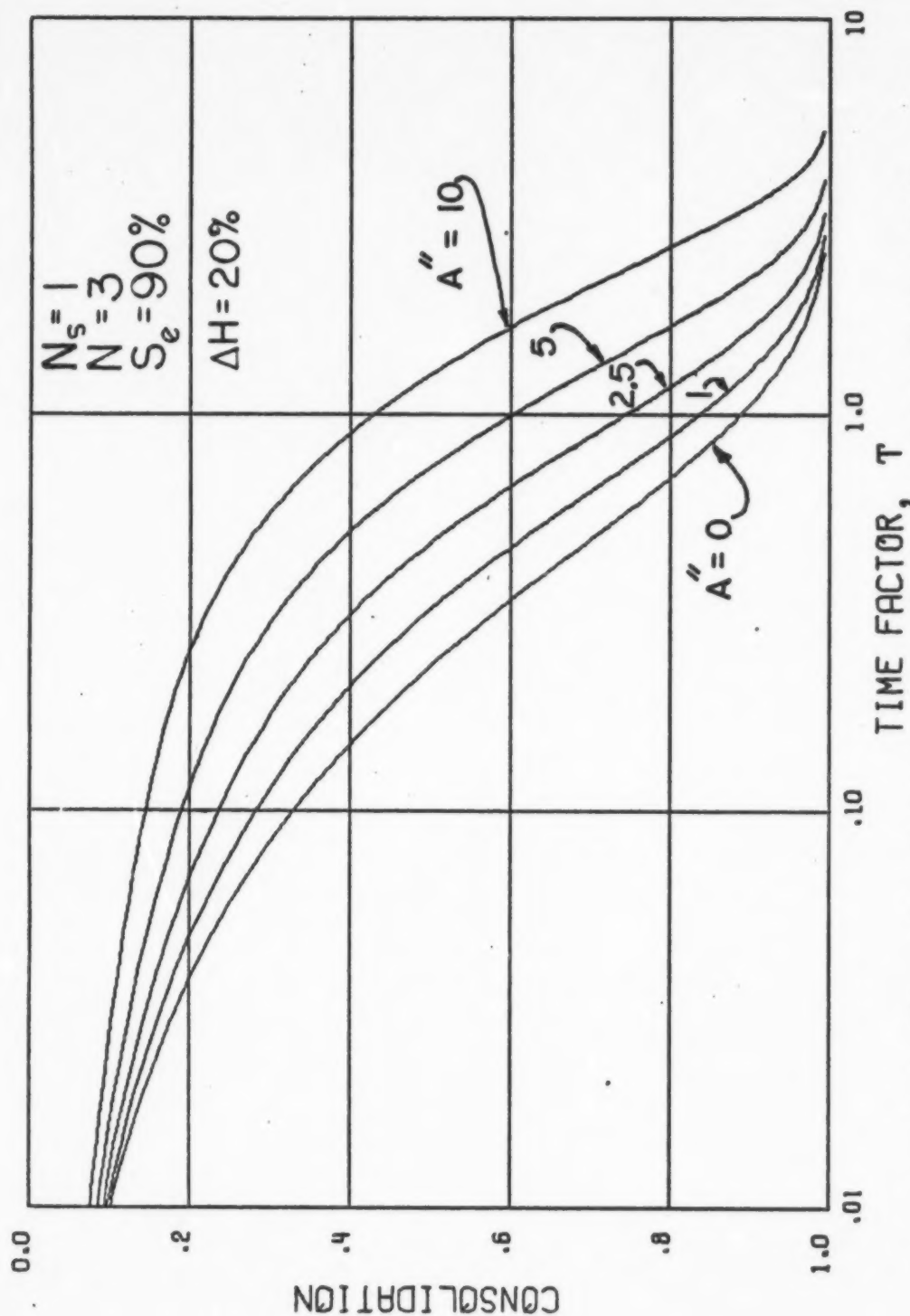


Figure 8-4: Time-settlement curves showing the effect of the variations in A'' upon the material behavior.

gases. As consolidation occurs, the pore pressures are reduced and the gas expands. The governing equation for this process is given by equation 8-1, which says that the change in volume due to fluid flow is a function of the amount of the fluid that flows from the sample, less the expansion of the gases present. Neither Barden (1965b) nor Toriyama and Sawada (1968) consider this effect. The importance of the saturation variation with time will depend on the power to which the effective saturation is to be raised, the magnitude of the load increase, and the initial degree of saturation.

The pressures used in the ideal gas laws, and thus in equation 5-49 are absolute, and therefore the unit loads used in the analysis must now be expressed in terms of percent of one atmosphere of pressure. Figure 8-5 shows the effect of the saturation variation and amount of load increase upon the behavior of otherwise dimensionally identical materials. The actual variations in effective saturation with time for the same case are shown in Figure 8-6. In Figure 8-5 the lower curve represents the response of the material with no saturation variation; corresponding to this is the straight line in Figure 8-6. With larger loads, the time required for the consolidation is greater and so is the variation in saturation with time. The effective saturation versus time plots terminate when the pore pressure becomes negligible. The three cases shown in Figures 8-5 and 8-6 all have 90 percent effective saturation immediately after loading; as would be expected, the greater the load and thus the greater the confining pressure, the greater the expansion of gas and the greater the variation in the saturation. The expansion of the gas reduces the transmissibility, delaying the fluid flow, and thus increasing the consolidation time.

The influence of the variation in the power of the effective saturation

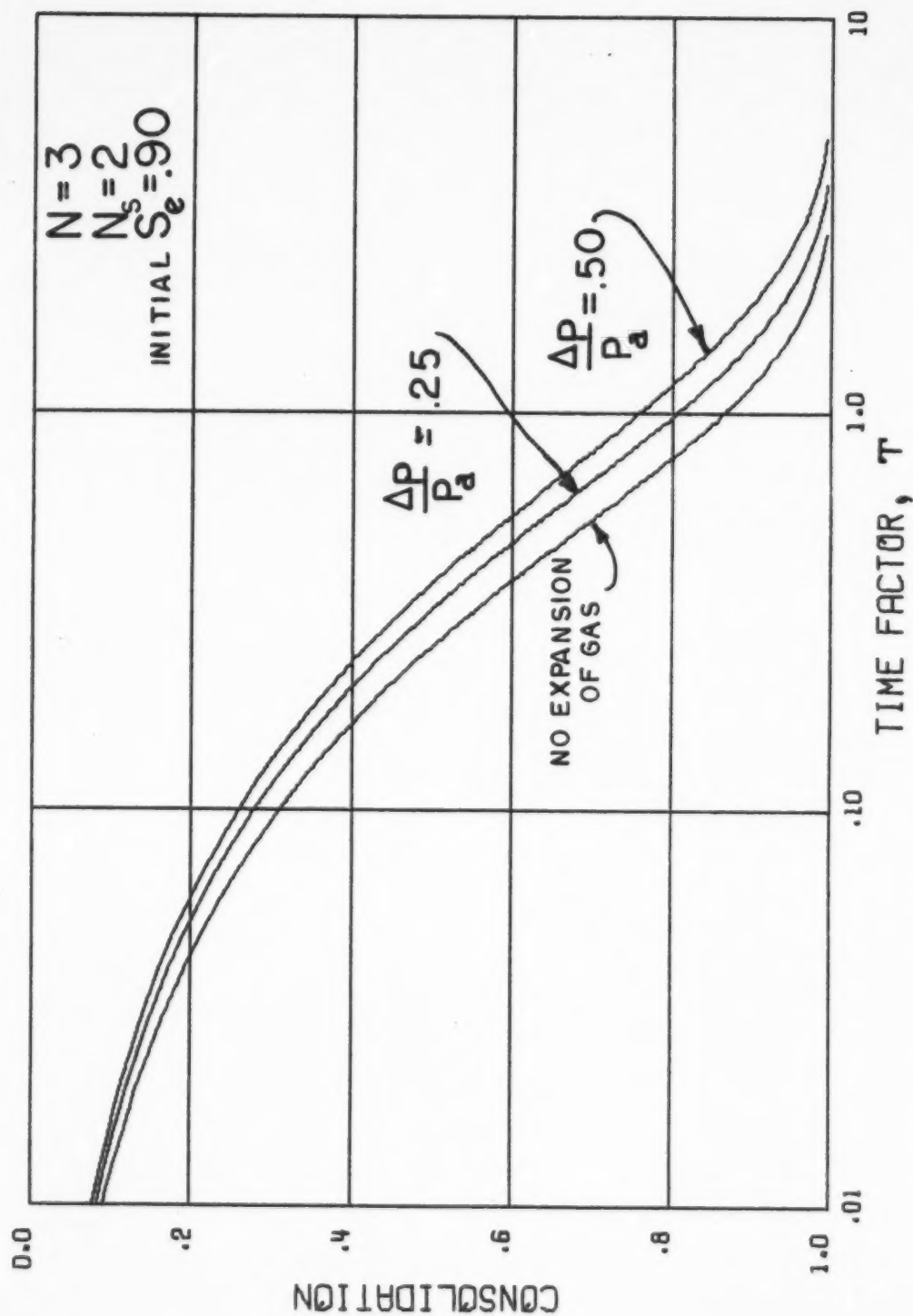


Figure 8-5: Time-settlement curves showing the effect of gas expansion and magnitude of load on the consolidation.

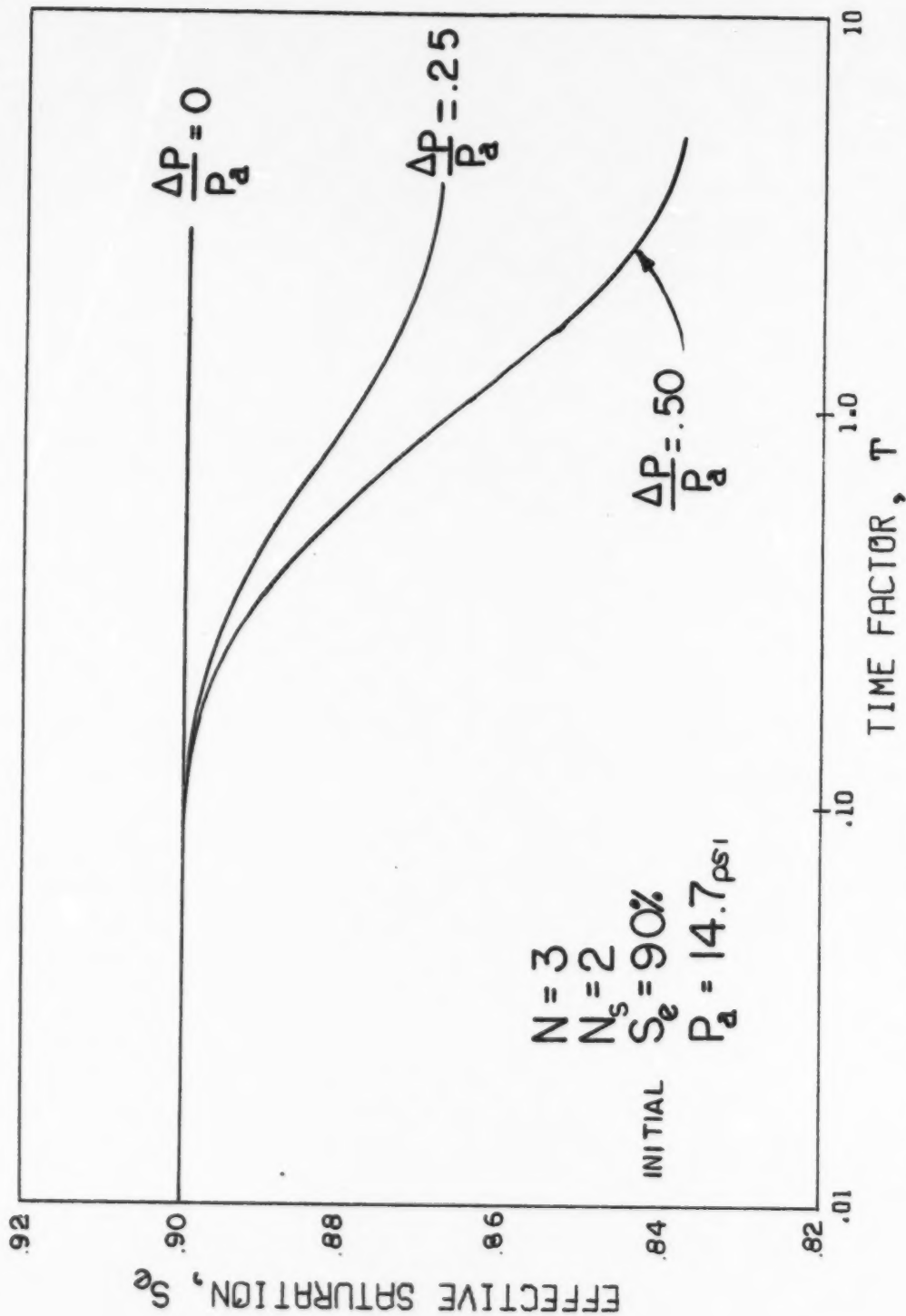


Figure 8-6: Time versus effective saturation curves showing the effect the magnitude of the load increase

is shown in Figures 8-7 and 8-8. Equation 8-1 is solved using the same properties as the material in Figure 8-1. The effect of the variation in N_s produces the same results as for Figure 8-1, but the effect of an additional term to account for variation of saturation with time is an additional delay in all the curves of Figures 8-7. The effective saturation versus time curves show the same kind of time delay as do the time-settlement curves with variation in the exponent N_s . The effect of the saturation after loading is shown in Figure 8-9, where the effective saturation varies from 100 to 60 percent. As the initial saturation decreases, the settlement-time graph moves to the right, or the time of consolidation increases.

Chemical and Biological Behavior

When relatively unstable organic materials are present in a mass, changes in their makeup will occur with time due to biological and/or chemical phenomena. Phase changes accompanying chemical and biological activity in the organic matter consist primarily of conversion of solids into liquid and vapor decomposition products. However, in terms of volume, the changes in the solid and liquid phases are insignificant compared to the gases generated. Volume changes from these effects in the solid and liquid phases will therefore be neglected in this study, and only changes in gas volume will be considered. Another effect of this behavior is that the solid matrix of the material will be directly or indirectly altered structurally, and the creep characteristics may be affected. No information is available concerning the effect of these changes on the material's properties; therefore the behavior of the solid matrix will be assumed to be unaffected by them. However, the changes that may occur could either

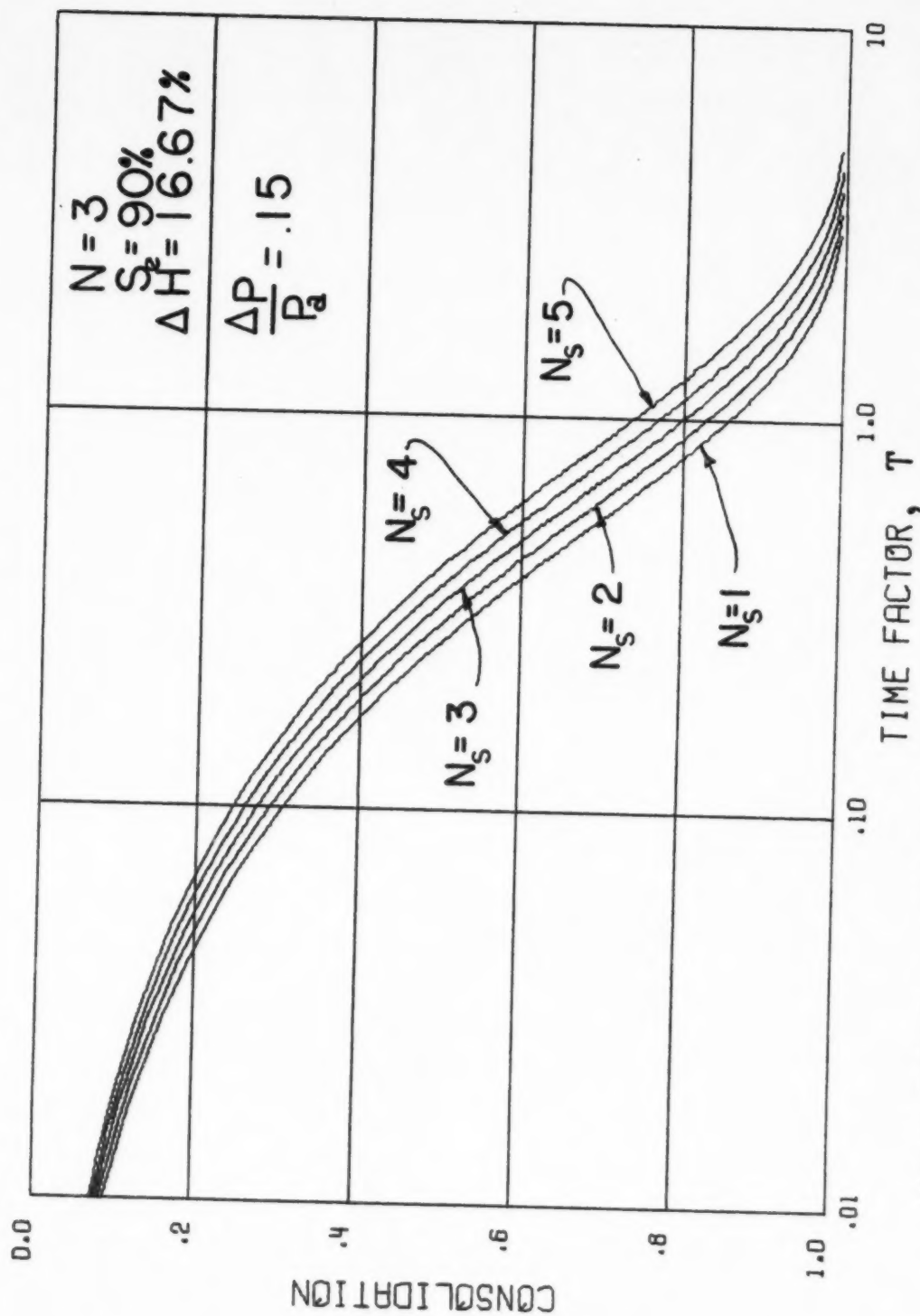


Figure 8-7: Time-settlement curves for variations of N_s , including effects of variation in saturation.

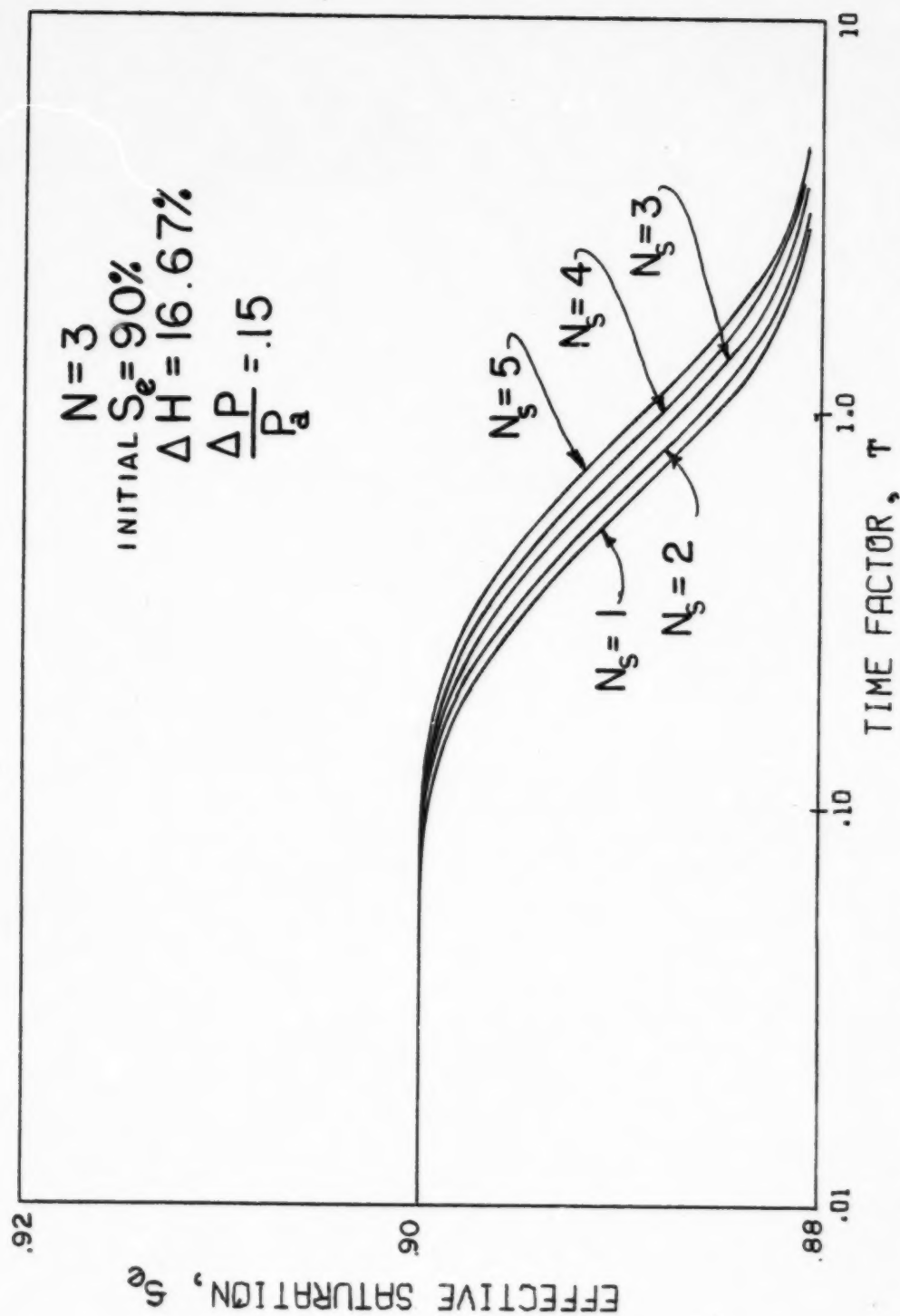


Figure 8-8: Time-effective saturation curves showing the effect on the variation of saturation with time for different values of N_s .

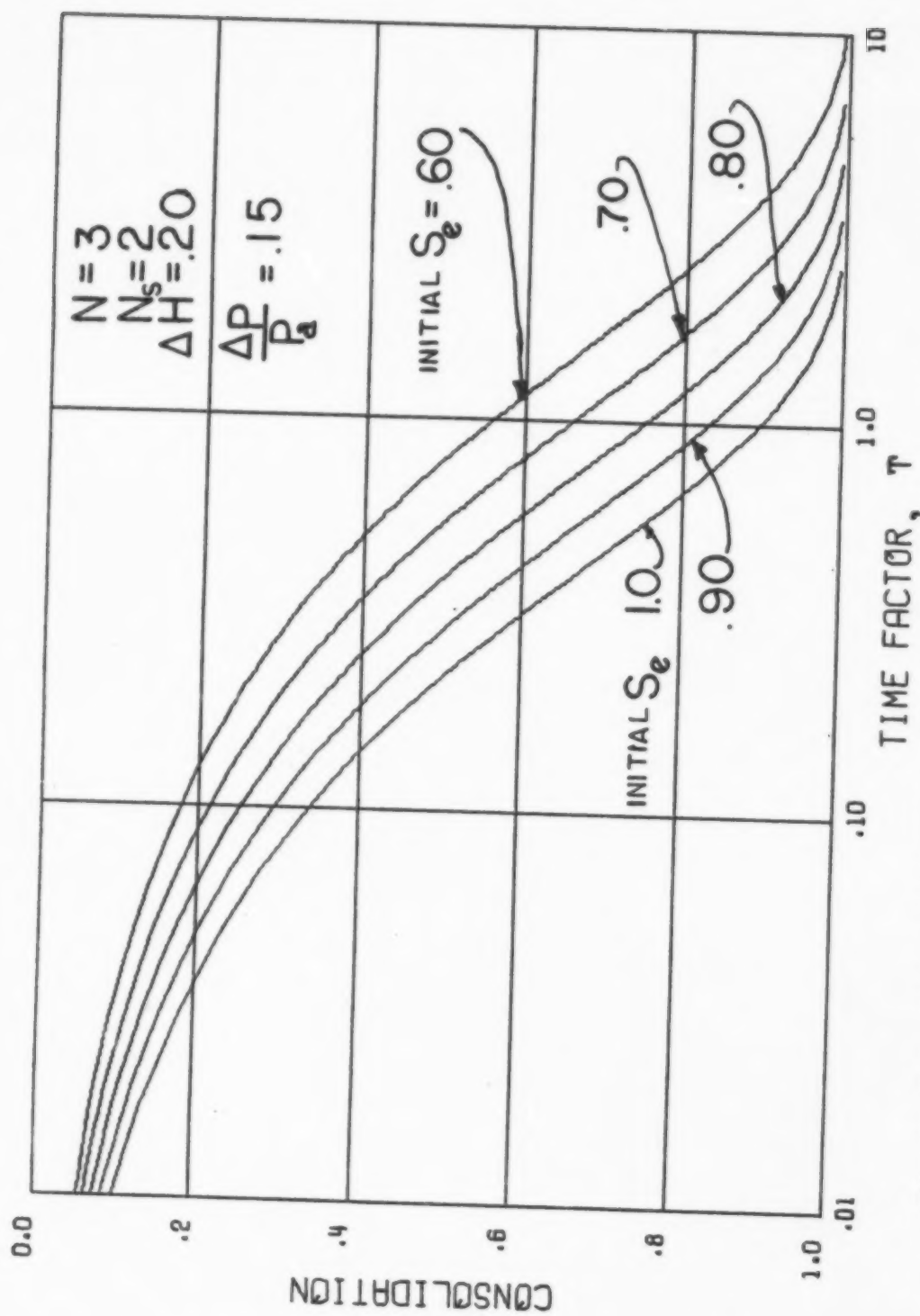


Figure 8-9: Time-settlement curves showing the effect of the initial effective saturation after loading.

increase or decrease the compressibility of the solid matrix.

The generation of gas tends to delay the movement of fluids and counterbalances the effect of previous fluid removal. Unfortunately, however, the nature of these processes is not well understood, and their time rates are undefined. Thus, the solution to the governing equation (6-10) will have to rely on arbitrary values of the parameters and the study of their effect is limited to relative changes. The generation of gas will produce a decrease in saturation and/or an increase in the pore pressure. If there is no fluid flow, or if the rate of production is large, the fluid pressure will increase due to the gas generation. If the pressure were unaffected by gas generation, it would have to be accompanied by fluid removal with a consequent decrease in saturation. The net results of this behavior is the retardation of the consolidation process. The magnitude and time variation of the gas production are important, since if the rate is great enough the consolidation process may be dominated by it. As the rate of gas production decreases, several interesting material responses develop. First, during the initial consolidation phase, before any fluid has left the inner voids, the fluid pressure at the mid-plane may become greater than that produced by the loading alone if the production of additional fluid is fast enough. Also, if the rate of gas production is great enough, the material will consolidate for some time and then the process will level off and the material may begin to expand. Unfortunately, the exact nature of the gas production is not known and it is unlikely that it is constant, but at the present time only a constant rate will be used.

These processes are accounted for in equation 6-10, which is given as

$$\frac{\partial \lambda}{\partial \tau} = S_e^N \frac{u_o}{\Delta p} \left(\frac{e}{e_o} \right)^N \left(\frac{1+e}{1+e_o} \right)^2 \left(\frac{\partial^2 u}{\partial z^2} + A' \left(\frac{\partial u}{\partial z} \right)^2 \right)$$

$$\frac{\partial u_o}{\partial \tau} \frac{H^2}{K_o(1+e_o)} \frac{(1+e)}{S} \bar{g}_{\gamma f} - \frac{e}{S} \frac{\partial S}{\partial \tau} \quad (8-3)$$

Figures 8-10 through 8-12 show the behavior of a material with the properties and conditions indicated and with varying rates of gas production. The mid-plane pore pressure versus time plot (Figure 8-10) shows that initially, the pore pressure will increase in the interior due to the fluid generation and if the fluid production is great enough, the settlement will stop prior to total stress transfer. Figure 8-11 shows that with a constant rate of gas production, the consolidation may never reach completion, and the material may even begin to swell. Negative gas generation rates (adsorption of gas) are also shown in these figures. As would be expected, this tends to speed the consolidation due to the additional fluid removal. Figure 8-12 shows the variation in saturation during the consolidation of the materials. As would be expected, the greater the rate of the production of gas, the greater the overall reduction in saturation. When the gas is adsorbed, the degree of saturation begins to increase after some period of time. This increase means that at some point in time the decrease in saturation due to the pressure removal becomes less than the increase due to adsorption.

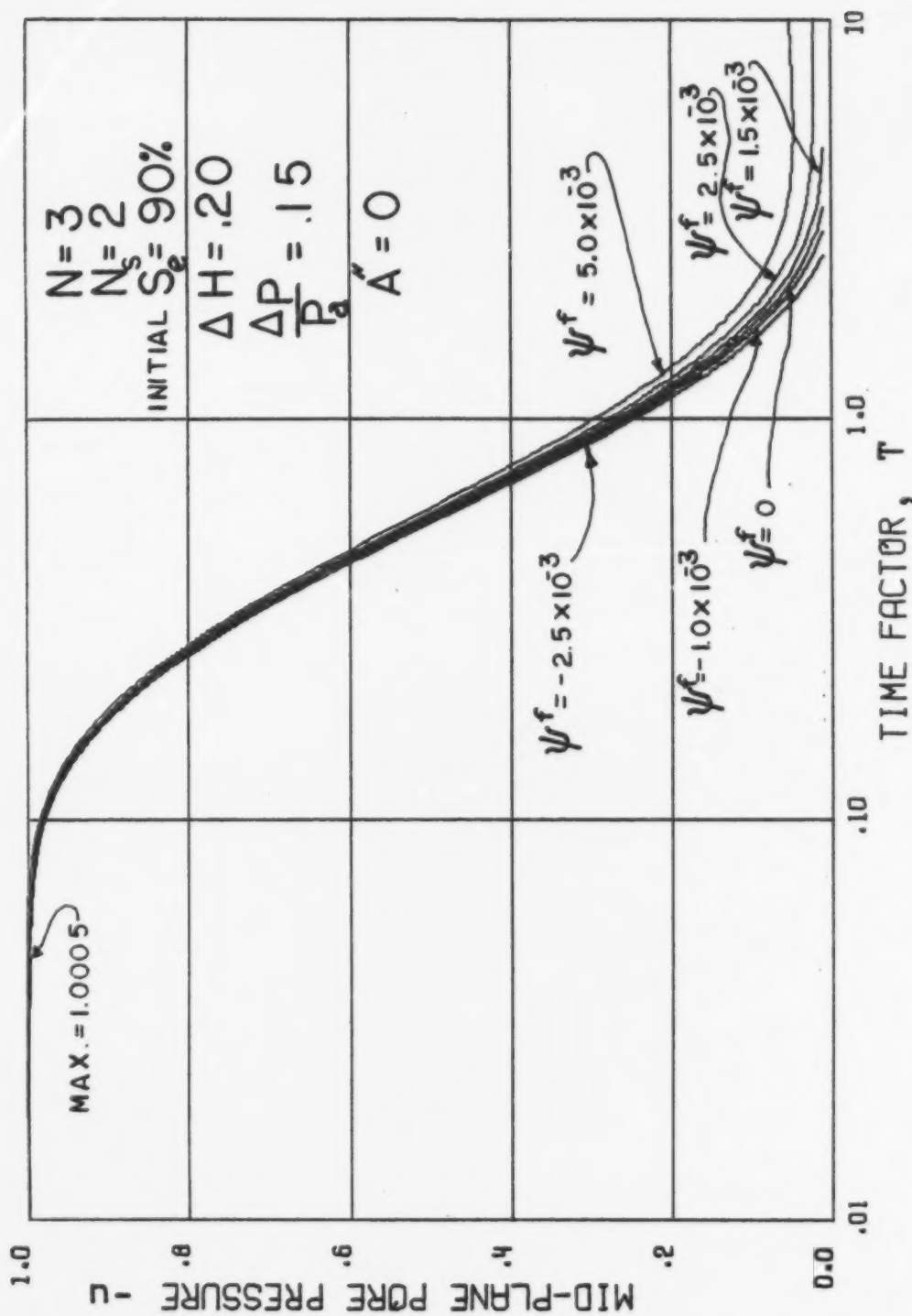


Figure 8-10: Time versus mid-plane pore pressure curves showing the effect of biological and/or chemical activity.

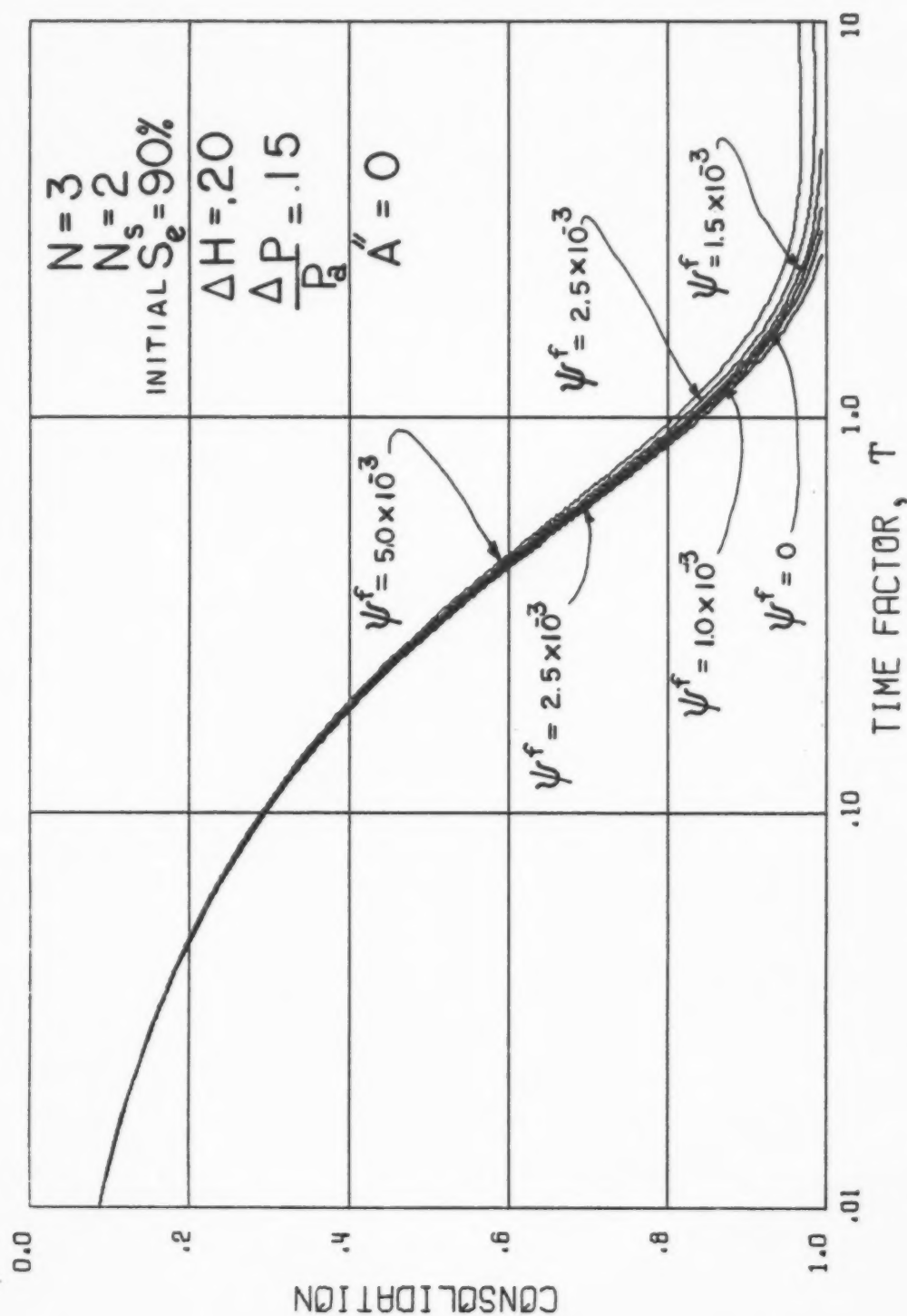


Figure 8-11: Time-settlement curves showing the effect of biological and/or chemical activity.

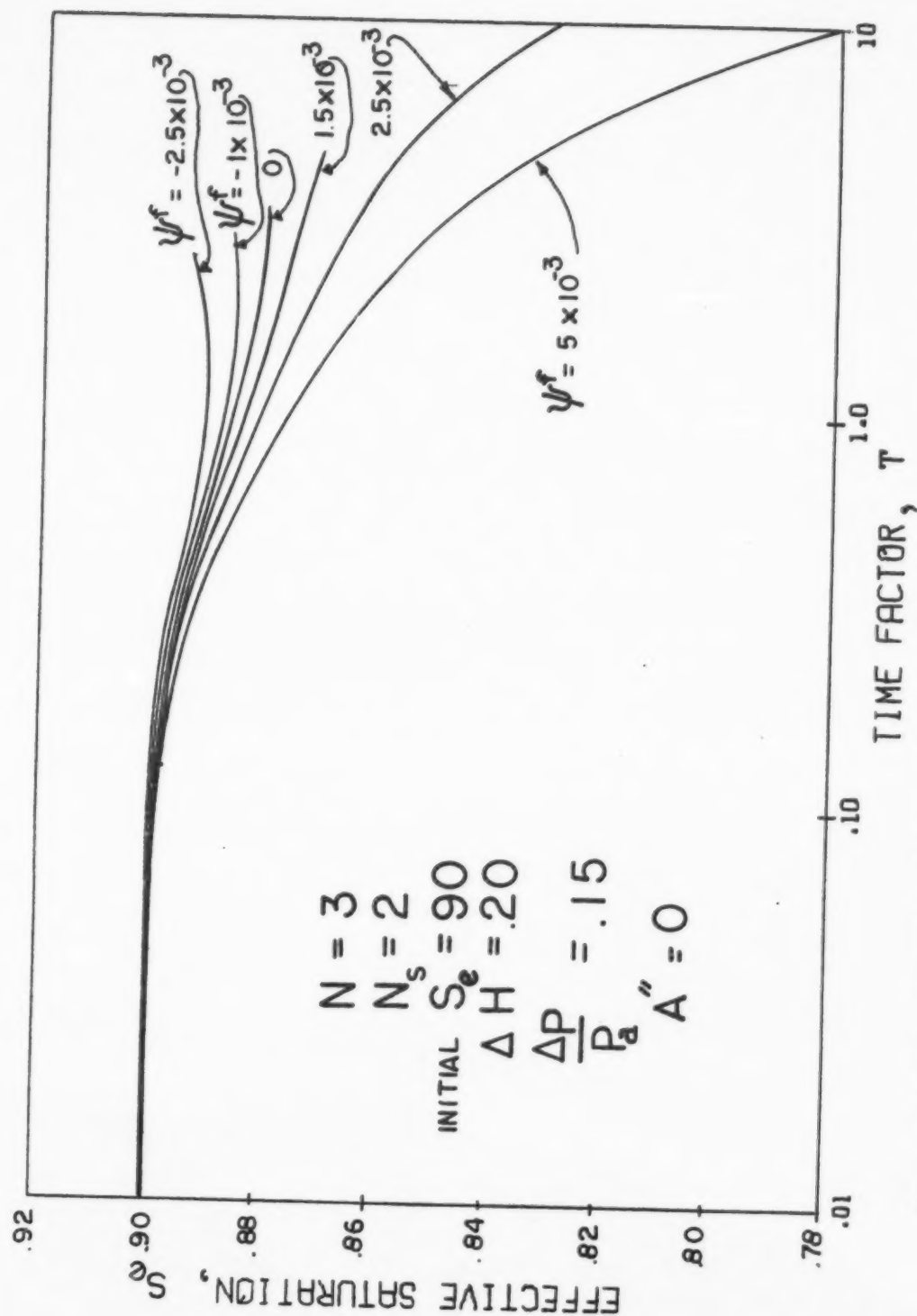


Figure 8-12: Time-effective saturation curves showing the effect of the biological and/or chemical activity.

Combined Model for the Unsaturated Case

The combined model involves the simultaneous solution of equations 6-16 and 8-3. In this case, the gas generation still occurs in what would otherwise be the pure creep phase, and must be considered. The equation describing the situation is

$$\frac{\partial \lambda}{\partial \tau} - \frac{1+e}{5} \gamma^f = \frac{1}{\Delta p \phi_c \tau_R} \sinh(\Delta p \phi_c) \quad (8-4)$$

The gas generation term produced negligible effects upon the behavior considered in this study, but if rate of activity were large enough, the process could be controlled by it. In general, the results of the analysis using the combined model could be predicted by the previous results, as can be seen in the following solutions.

Figures 8-13 and 8-14 show the results of the combined analysis for the case of no gas generation during consolidation. The four curves represent initial effective saturation values of 100, 90, 70 and 50 percent, and show that the consolidation and pore pressure dissipation are delayed by the decreasing saturation.

The influence of the variation in saturation due to the reduction of the excess fluid pressure is shown in Figures 8-15 and 8-16 where the lower curve shows the behavior of the material with no saturation variation. All three cases begin with an effective saturation of 90 percent; however, the center curve is initially loaded with one-tenth of an atmosphere while, for the upper curve, excess fluid pressure is equal to one-quarter of an atmosphere. As before, the greater the load, the slower the consolidation.

The effect of the production or adsorption of gas is shown in Figures

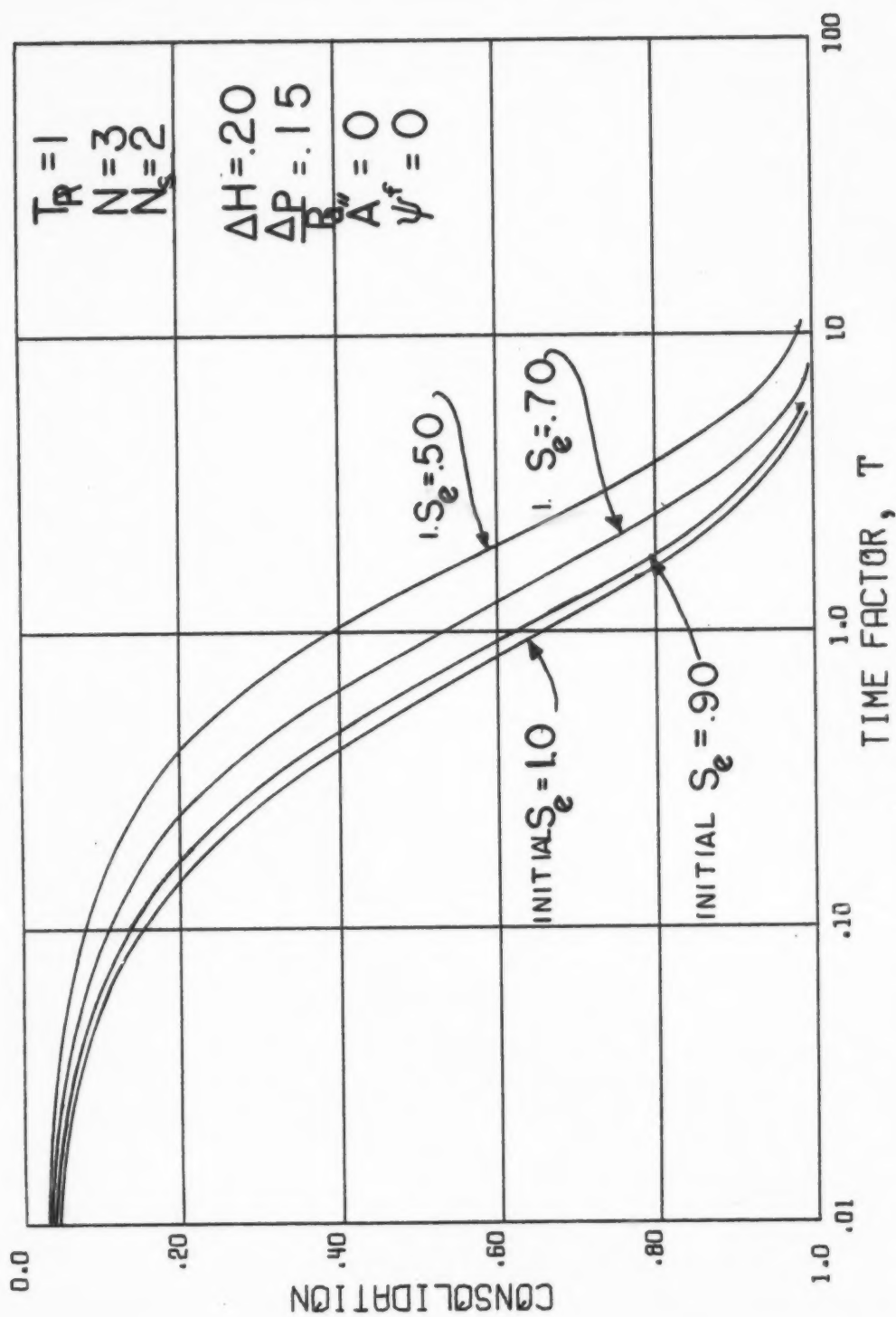


Figure 8-13: Time-consolidation curves for the combined model showing the effect of variations in effective saturation.

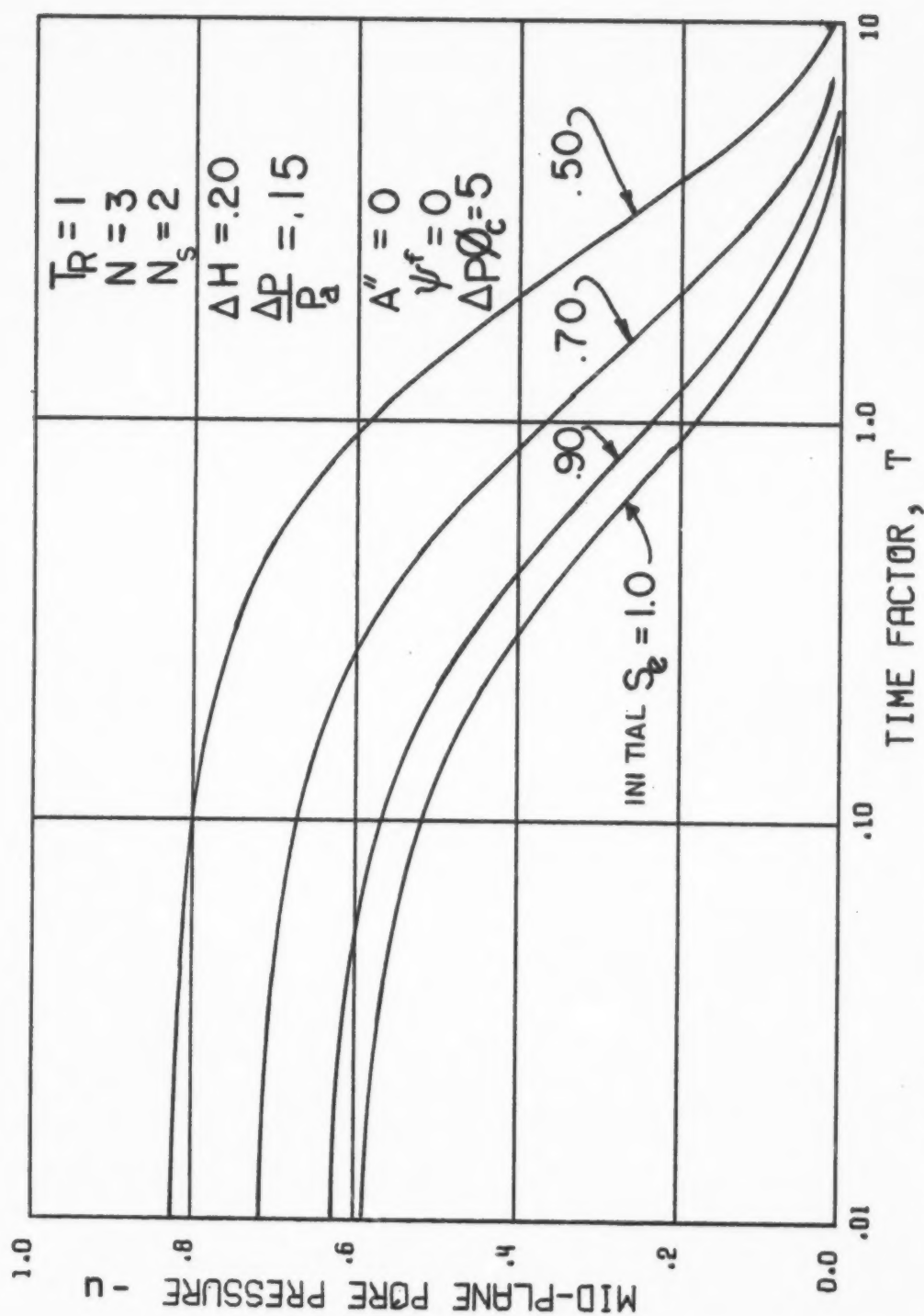


Figure 8-14: Time-pore pressure curves for the combined model showing the effects of the variations of the effective saturation.

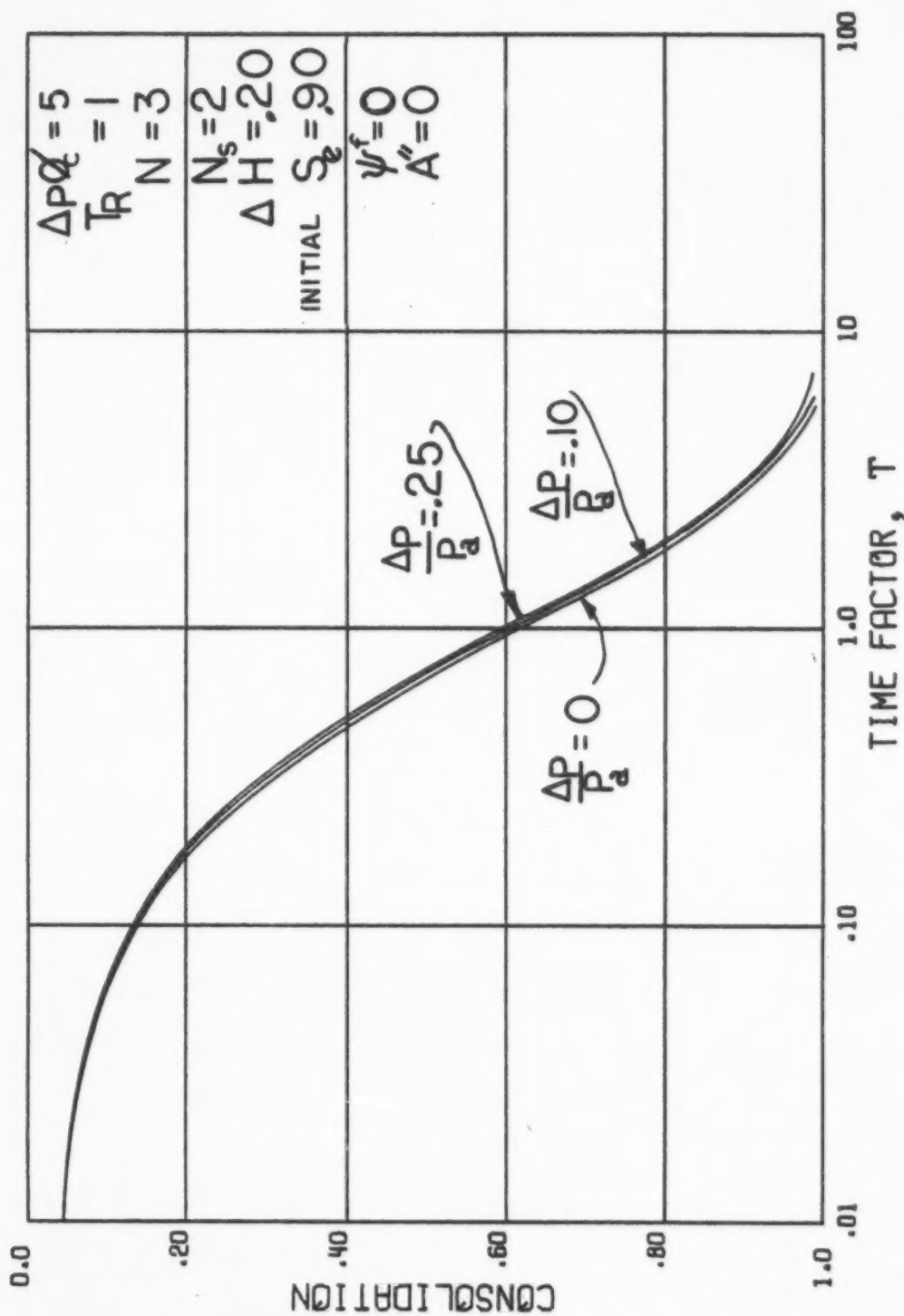


Figure 8-15: Time-settlement curves for combined model showing the effect of variations in the magnitude of load and saturation.

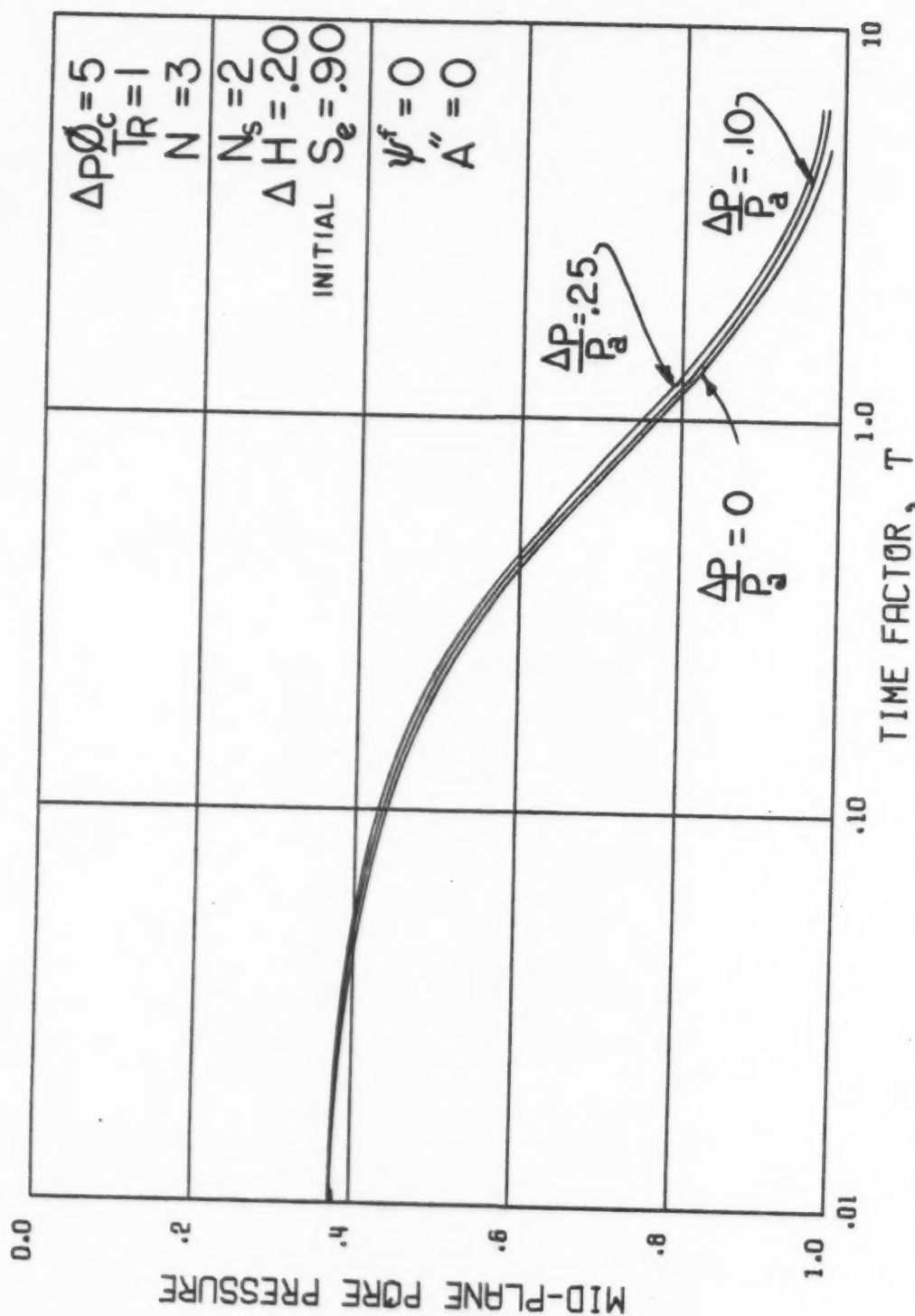


Figure 8-16: Time versus mid-plane pore pressure curves for combined model showing the effects of variations in the magnitude of load and saturation.

8-17, 8-18, and 8-19, where the production and adsorption rates are both 2.5×10^{-3} unit volumes per unit time. As before the gas generation decreases the speed of consolidation and pressure dissipation, while adsorption increases their rates. The center curve represents no fluid source or sink in the material. The consolidation curve for the increase in fluid is terminated at a dimensionless time of ten because of instability in the combined model when the fluid lost became equal to that produced.

In Chapter VII, some of the graphs showed that as the structural-pore pressure time ratio increased, the deviations due to the different variables were damped out by the influence of the creep. Figures 8-20 and 8-21 show the effect of the saturation variations and gas generation for a time ratio of five. The two lower curves in the consolidation and pore pressure versus time graphs represent the action of materials with constant saturations throughout the process. The lower curve is for the case of totally saturated behavior while the middle one has a constant effective saturation of 90 percent. The two cases exhibit nearly identical consolidation-time curves, while their pore pressure-time curves tend to merge after some period of time. The upper curves in these two graphs represent a material with an initial effective saturation of 90 percent, and a load of two-tenths of one atmosphere, with a gas production of .0015 unit volume/unit time. As before, the model became unstable after most of the fluid pressure was dissipated. The upper curve does indicate that near the termination of the pore pressure phase, the biological activity tends to level out the deflections as shown before in the no-creep model.

The increasing influence of the creep with the increase in the time ratio is shown in Figures 8-22 and 8-23 where the consolidation for time

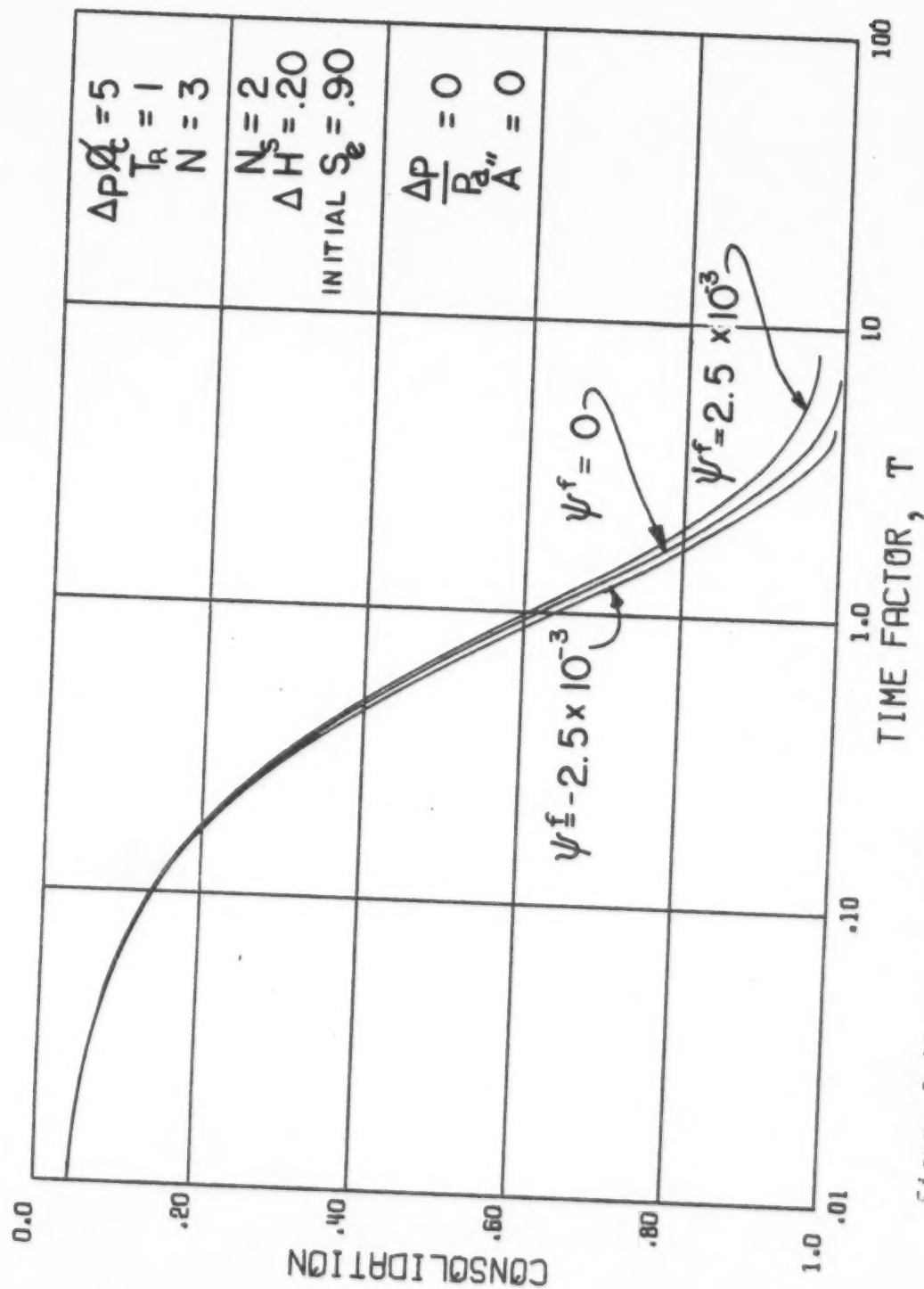


Figure 8-17: Time-consolidation curves for combined model showing the effects of gas generation and adsorption

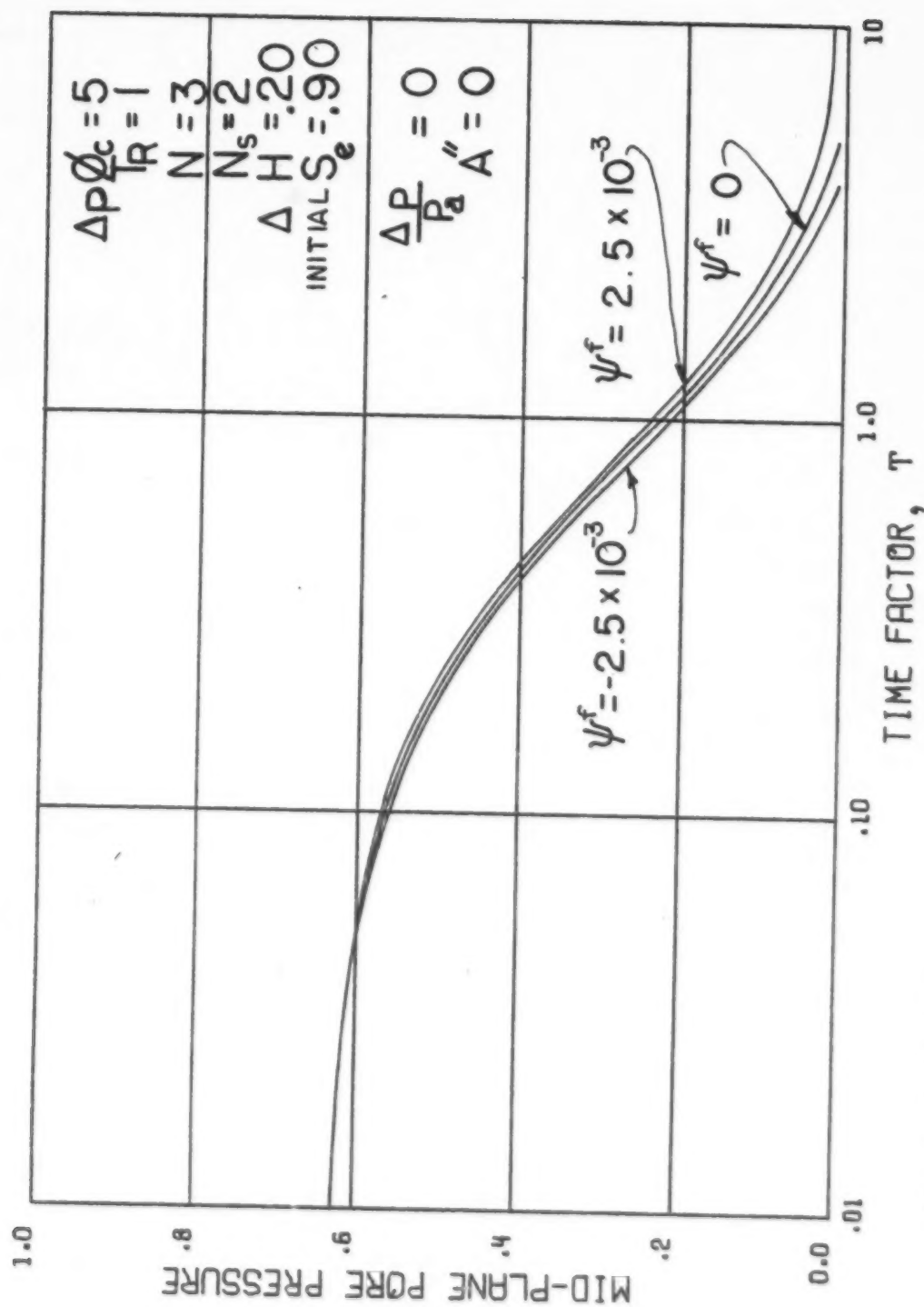


Figure 8-18: Time - mid-plane pore pressure for combined model showing the effects of gas generation and adsorption.

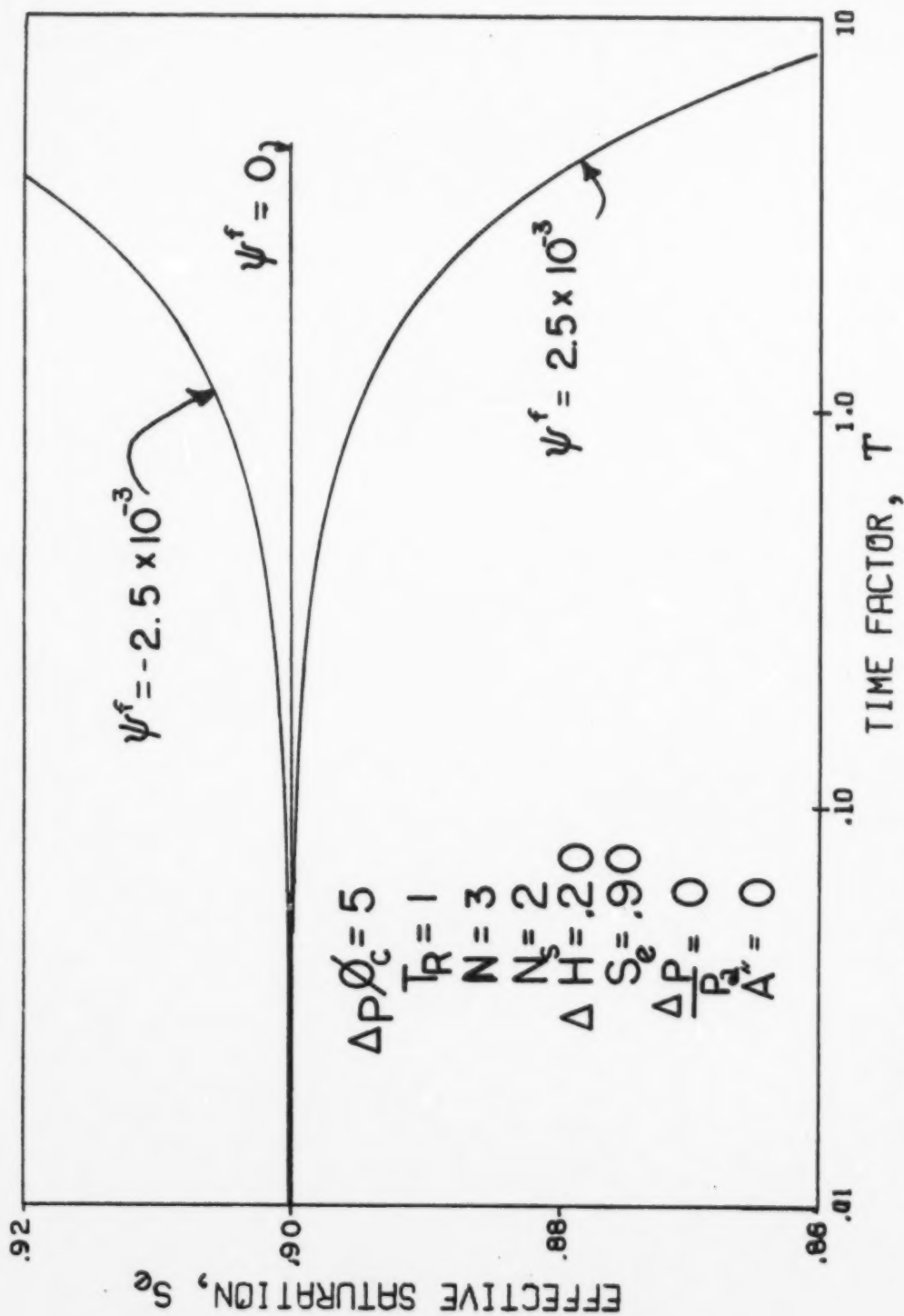


Figure 8-19: Time-effective saturation curves for combined model showing the effects of gas generation and adsorption.

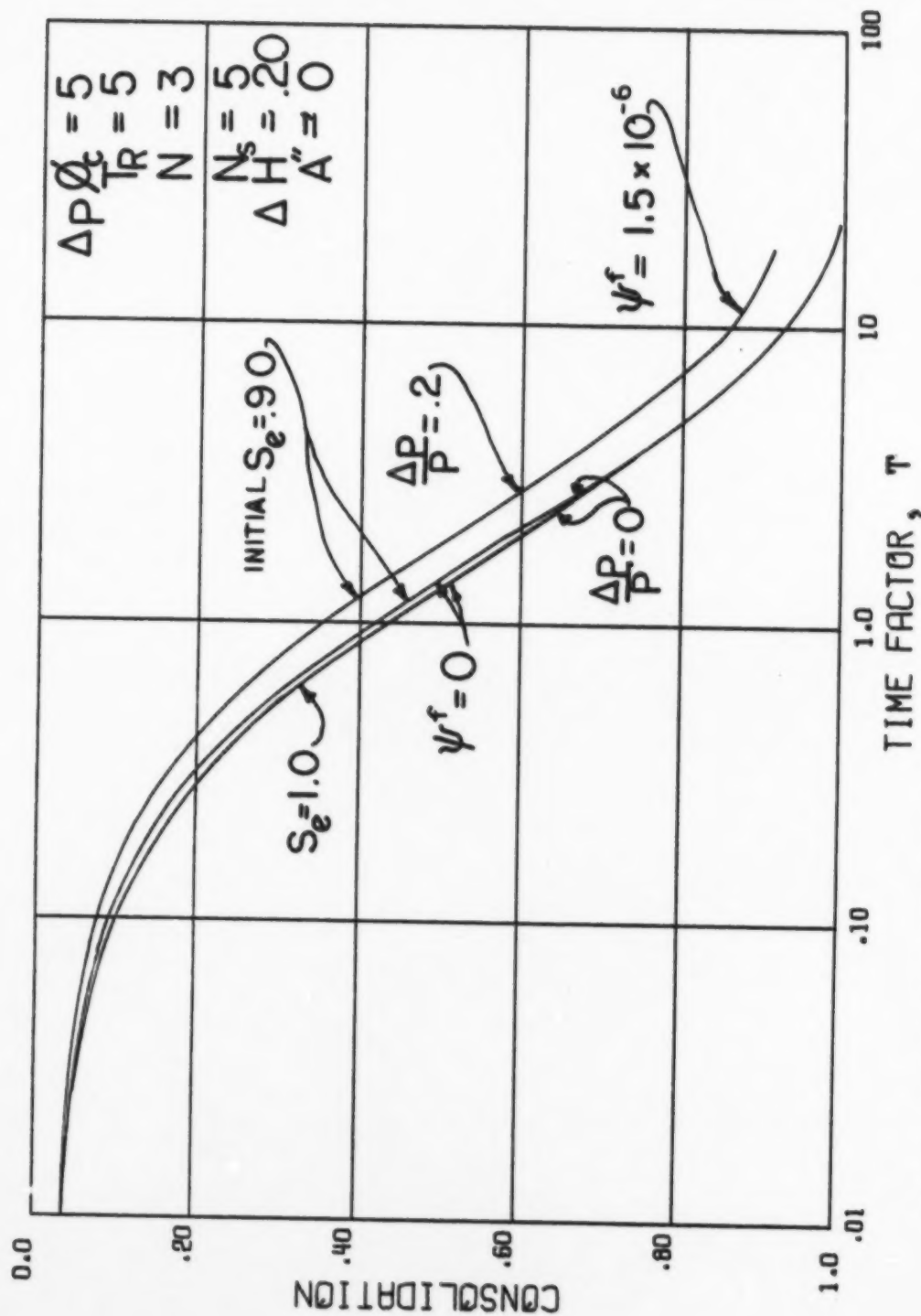


Figure 8-20: Time-consolidation curves showing the dual effect of the variations in saturation and fluid generation upon the behavior of the combined model.

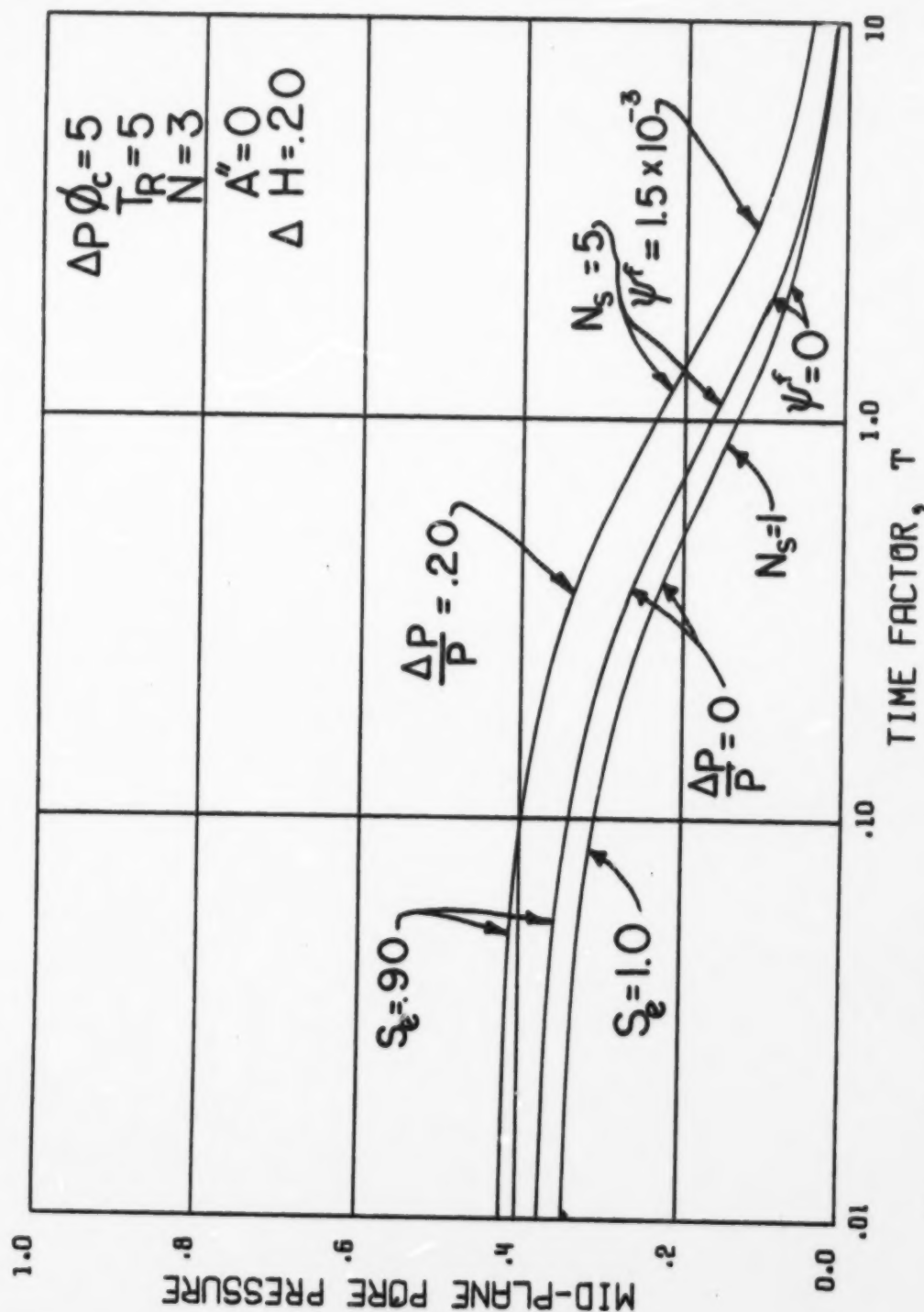


Figure 8-21: Time versus mid-plane pore pressure curves for the combined model showing the dual effects of variations in saturation and fluid generation.

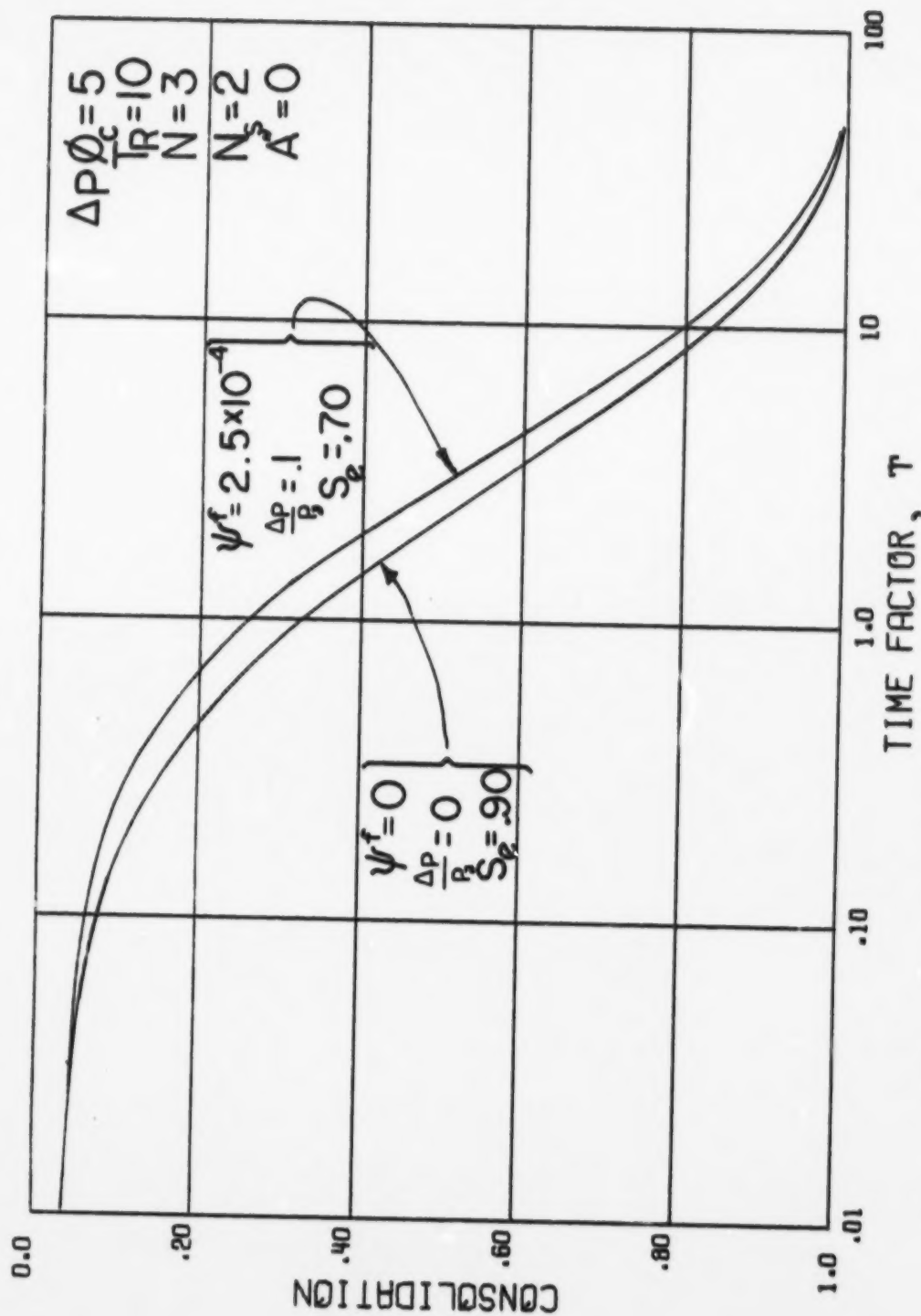


Figure 8-22: Time-consolidation curves for a time ratio of ten, showing the decreasing effect of gas generation and saturation with increasing time ratio.

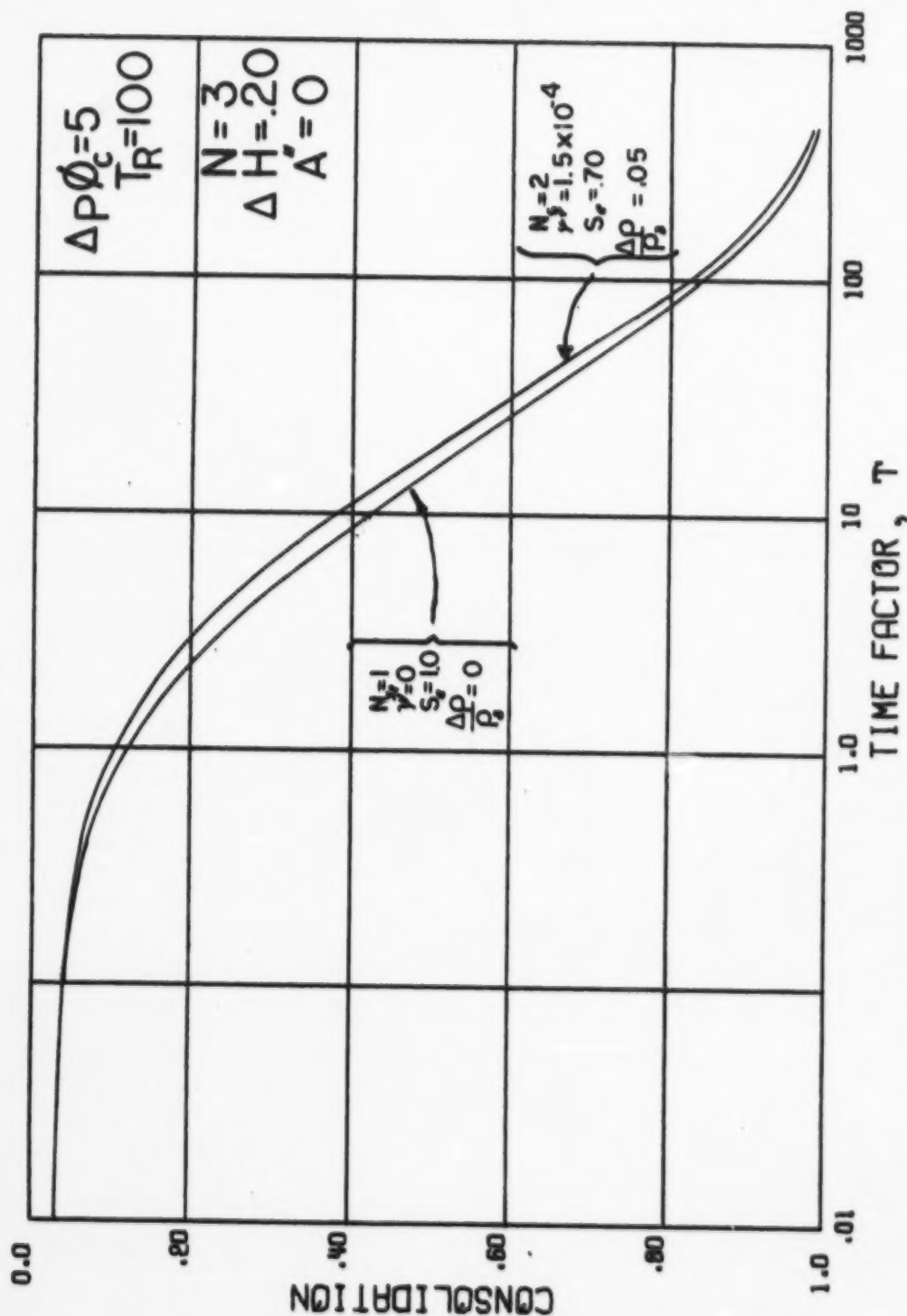


Figure 0-23: Time-consolidation curves for a time ratio of one hundred showing the decreasing influence of load and gas generation with time ratio increase

ratios of ten and 100 are shown. The lower curve in the first of the two figures represents the behavior of an inert material that has an initial effective saturation of 90 percent, while the other curve has an initial effective saturation of 70 percent, load equal to one-tenth of an atmosphere, and fluid production equal to 2.5×10^{-4} unit volume/unit time. The two consolidation curves begin and end at nearly identical times, while the one with the varying saturation and with gas production lags behind the other one throughout most of the process. However, the production rate of gas is smaller for this case than in previous ones due to the instabilities in the model when the larger values are used. The behavior is similar, although not as pronounced as that in Figure 8-20 where the rate of consolidation tends to zero with a positive pore pressure still remaining. Figure 8-23 shows a similar behavior to 8-22 for a model with higher time ratio (100). The lower curve represents a saturated, inert material with an initial effective saturation of 90 percent. The upper curve is for material with 70 percent initial effective saturation, initial load increase of one-twentieth of one atmosphere, and a gas generation rate of 1.5×10^{-4} units of volume/unit of time. These curves exhibit behavior, although to a lesser degree, similar to that shown in Figure 8-22. As before, the curves part, then nearly merge, although the upper curve tends to remain somewhat above the lower one due to the generation of the gas.

As with the dimensionless time, the gas production rate must be put in relative terms. If it is assumed that 25 percent of the total weight of refuse will be converted to gas, the specific gravity of the material is 1.5, the void ratio is ten, and that the gas production rate is

1.4×10^{-3} dimensionless volume/dimensionless time, and the method of analysis of Hermann and Quon (1970) is used, the total time required for decomposition at a steady rate is 2×10^4 unit times. If a unit time is one day, then about 50 years would be required for total breakdown.

Summary

In this chapter, the effects of partial saturation, changes in saturation, and gas generation during consolidation upon the behavior of the material are discussed. Initially, the delay of consolidation due to the presence of gas in the pore fluid and how this delay varies with the power of the effective saturation relationship are investigated. Later, the additional delay caused by the gas expansion is shown. The effects of the generation and adsorption of gas on the settlement rate are shown in Figure 8-10. The rate of settlement of the material decreases as the rate of gas generation increases, and settlement may stop after some time if enough gas is evolved. On the other hand, adsorption of gas will increase the rate of consolidation.

When the unsaturated fluid model is combined with the creep function, the variations in the time-settlement curves are not as great as in the no-creep model. However, the magnitude of this effect is not as great for the unsaturated case as for the saturated. For large time ratios, the time-settlement curves tend to coincide at the beginning and end of consolidation but diverge in the middle. Thus partial saturation and its variations during consolidation will produce time delays that depend upon the initial saturation after the load is applied, the stiffness of the solid matrix, the amount of creep and rate of pore fluid changes.

CHAPTER IX

SUMMARY AND CONCLUSIONS

Summary

The time-settlement behavior of processed refuse is modeled and investigated in this study. Two simultaneous partial differential equations which include the effects of finite strains, biological and chemical activity, variations in saturation, and a large amount of creep displacement are used to simulate the material response. These equations are combined with proposed functions that relate the instantaneous geometry to the variation in material properties such as permeability and compressibility. The influence of the different variables on the response of each equation is examined, as well as on their simultaneous behavior.

The general make-up and behavior of urban refuse is presented in Chapter II. The biological and chemical processes that occur are discussed along with certain mechanical considerations. Chapter III deals with the advantages and disadvantages of milling refuse compared with conventional disposal methods. An analogy between the behavior of peats and that of milled refuse is made in Chapter IV. The general time-settlement model is also developed in this chapter. The interrelationships of permeability and pore geometry, transmissibility and saturation, compressibility and void ratio, and partial saturation and load distribution are investigated in Chapter V. The influence of the different parameters upon the behavior of both mechanisms of settlement is studied in Chapter VI, using the proposed functions of Chapter V. The results of this study are presented in graphical form as time-settlement curves. The simultaneous effects of each mechanism of settlement is examined in Chapter VII. Chapter VIII deals

with the effects of partial saturation and phase changes on the two settlement mechanisms simultaneously.

Conclusions

Within the framework of the study described herein for the consolidation of milled refuse, the following conclusions can be advanced:

1. The model proposed represents the time-settlement behavior of a material with void ratios greater than unity, saturation, compressibility, and permeability varying with time, while undergoing finite strains and biochemical decomposition. When it is assumed that there is total saturation, that the compressibility-permeability ratio is constant, that strains are small, and that no biochemical changes occur, the model reduces to the classical Terzaghi case.
2. A very good fit is obtained between the data available and proposed functions for the variation of the properties of peats. The changes in the compressibility and permeability with those of geometry fit the empirical relationships quite well.
3. For the mechanism of pressure dissipation only, the effect of including the influence of both non-linear variation of fluid pressure with depth and variation of the compressibility-permeability ratio with time is to increase the predicted duration of settlement. This retardation of the predicted response depends on the magnitude of the coefficient of the non-linear term, which in principle must be found from laboratory tests, and on the increase of the compressibility ratio. The predicted duration may, for the range of parameters investigated, be greater than 30 times that predicted by the classical theory.

4. The relative influence of the two settlement mechanisms (pore pressure dissipation and creep) is a function of the amount of load increase. For small load changes ($\Delta p/p \leq 0.5$), creep will dominate the predicted response while for large changes ($\Delta p/p \geq 1.0$), the pressure dissipation mechanism will dominate. For intermediate cases, the response will be a composite of the two mechanisms.
5. The amount of increase of the model's response time due to variations in the parameters depends on which mechanisms are included. The effects of the non-linearity and finite strains are less apparent when both processes are considered, and may be nearly obscured when the creep mechanism is dominant.
6. The rate of response of the unsaturated model can vary greatly, depending largely on the degree of saturation. If saturation is below the residual value, only gas will flow, and the rate of settlement will be controlled by creep. On the other hand, for a case when saturation is greater than the residual, the fluid pressure dissipation will also affect the behavior. In this case, the pressure dissipation is hindered by the presence of gas which may block the fluid flow channels. Also the expansion of the gas due to the relief of the fluid pressures tends to delay consolidation.
7. Production of gas and/or pore fluid will cause a delay in the settlement response, and may even dominate the material's behavior. If gas is adsorbed, however, the consolidation rate will increase.
8. The increase in response times that result from unsaturated conditions is obscured to a lesser degree than that of the non-linear term and coefficient containing void ratios.

BIBLIOGRAPHY

- Adams, J.I., "The Engineering Behavior of a Canadian Muskeg," Proceedings of the Sixth International Conference on Soil Mechanics and Foundation Engineering, Montreal, Vol. I, 1965, pp. 3-7.
- Arman, A., A Definition of Organic Soils, Engineering Research Bulletin No. 101, Louisiana State University, 1969.
- Astrojet-General Corporation, California Waste Management Study, Report No. 3056, Azusa, California, 1965.
- Atchison, G.D., "Relationships of Moisture Stress and Effective Stress Functions in Unsaturated Soils," Pore Pressure and Suction in Soils, Butterworth, 1960, pp. 47-52.
- Barden, L., "Consolidation of Clay with Non-linear Viscosity," Geotechnique, Vol. 15, No. 4, 1965a, pp. 345-362.
- Barden, L., "Consolidation of Compacted and Unsaturated Clay," Geotechnique, Vol. 15, No. 3, 1965b, pp. 267-286.
- Barden, L., "Primary and Secondary Consolidation of Clay and Peat," Geotechnique, Vol. 18, No. 1, 1968, pp. 1-24.
- Barden, L., "Time Dependent Deformation of Normally Consolidated Clays and Peats," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 95, No. SMI, 1969, pp. 1-31.
- Barden, L., and Berry, P.L., "Model of the Consolidation Process in Peat Soils," Proceedings of the Third International Peat Congress, Quebec, 1968, pp. 119-125.
- Barden, L., Madeador, A., and Sides, G.R., "Volume Change Characteristics of Unsaturated Clay," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 95, No. SMI, 1969, pp. 33-50.
- Bird, R.B., Stewart, W.E., and Lightfoot, E.W., Transport Phenomena, John Wiley and Sons, Inc., New York, 1966.
- Bishop, A.W., "Some Factors Controlling the Pore Pressure Setup during the Construction of Earth Dams," Proceedings of the Fourth Conference on Soil Mechanics and Foundation Engineering, London, Vol. II, 1957, pp. 294-300.
- Bishop, A.W., "The Measurement of Pore Pressure in the Triaxial Test," Pore Pressure and Suction in Soils, Butterworth, 1960a, pp. 38-46.
- Bishop, A.W., Discussion, Pore Pressure and Suction in Soils, Butterworth, 1960b, pp. 63-66.

- Bishop, A.W., Alpan, I., Blight, G.E., and Donald, I.B., "Factors Controlling the Strength of Partly Saturated Cohesive Soils," ASCE Research Conference on Shear Strength of Cohesive Soils, Boulder, 1960, pp. 503-532.
- Bishop, A.W., and Bjerrum, L., "The Relevance of the Triaxial Test to the Solution of Stability Problems," ASCE Research Conference on Shear Strength of Cohesive Soils, Boulder, 1960, pp. 437-501.
- Bishop, A.W., and Blight, G.E., "Some Aspects of Effective Stress in Saturated and Partly Saturated Soils," Geotechnique, Vol. 13, No. 3, 1963, pp. 177-197.
- Bishop, A.W., and Donald, I.B., "The Experimental Study of Partly Saturated Soil in the Triaxial Apparatus," Proceedings of the Fifth International Conference on Soil Mechanics and Foundation Engineering, Paris, Vol. I, 1961, pp. 13-21.
- Bishop, A.W., and Eldin, G., "Undrained Triaxial Tests on Saturated Sands and Their Significance in the General Theory on Shear Strength," Geotechnique, Vol. 2, No. 1, 1950, pp. 13-32.
- Bishop, A.W., Kennard, M.F., and Vaughan, P.R., "Developments in the Measurement and Interpretation of Pore Pressure in Earth Dams," Eighth International Congress on Large Dams, Edinburgh, Vol. II, 1969, pp. 47-72.
- Brutsaert, W., "On Pore Size Distribution and Relative Permeability of Porous Mediums," Journal of Geophysical Research, Vol. 68, No. 8, 1963, pp. 2233-2235.
- Brutsaert, W., "Probability Laws for Pore Size Distribution," Soil Science, Vol. 101, No. 2, 1966, pp. 85-92.
- Brutsaert, W., "The Permeability of a Porous Medium Determined for Certain Probability Laws of Pore Size Distribution," Water Resources Research, Vol. 4, No. 2, 1968, pp. 425-434.
- Carnahan, B., Luther, H.A., and Wilkes, J.O., Applied Numerical Methods, John Wiley and Sons, Inc., New York, 1969.
- Carpenter, W.L., Vamvakias, J.G., and Gellman, I., "Temperature Relationships in Aerobic Treatment and Disposal of Pulp and Paper Waste," Journal of the Water Pollution Control Federation, Vol. 40, No. 5, 1968, pp. 733-740.
- Childs, E.C., and George, N.C., "Soil Geometry and Soil-Water Equilibria," Discussions of the Faraday Society, Vol. 3, 1948, pp. 78-85.
- Childs, E.C., and George, N.C., "The Permeability of Porous Materials," Proceedings of the Royal Society, London, Vol. 201A, 1950, pp. 392-405.

- Christensen, R.W., and Wu, T.H., "Analysis of Clay Deformation as a Rate Process," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 90, No. SM6, 1964, pp. 125-157.
- Collis-George, N., "Relationship between Air and Water Permeability in Porous Media," Soil Science, Vol. 76, No. 4, 1953, pp. 239-251.
- Croney, D., and Coleman, J.D., "Pore Pressure and Suction in Soil," Pore Pressure and Suction in Soils, Butterworth, London, 1960, pp. 31-37.
- Davis, E.H., and Raymond, G.P., "A Non-linear Theory of Consolidation," Geotechnique, Vol. 15, No. 2, 1965, pp. 161-173.
- De Marco, J., Keller, D.J., Newton, J.L., and Leckman, J., Incinerator Guidelines--1969, U.S. Department of Health, Education and Welfare, Public Health Service, Bureau of Solid Water Management, Cincinnati, 1969.
- Dobson, A.L., Microbial Decomposition Investigations in Sanitary Landfills, Ph.D. Dissertation, West Virginia University, Morgantown, 1964.
- Donald, I.B., Discussion, Pore Pressure and Suction in Soils, Butterworth, 1960, pp. 69-70.
- Douglas, J., "A Numerical Method for the Solution of a Parabolic System," Numerische Mathematik, Vol. 2, No. 2, 1960, pp. 91-98.
- Dunn, W.L., "Refuse Filling of a Swampland in a Fresh Water Lake," Civil Engineering, Vol. 38, No. 1, 1968, pp. 60-61.
- Eckenfelder, W.W., Industrial Water Pollution Control, McGraw-Hill Book Company, New York, 1966.
- Fagan, D., An Investigation of the Permeability of Compressed Municipal Refuse and the Pollution Concentration of the Resultant Leachings, M.S. Thesis, Northwestern University, 1971.
- Fair, G.M., Geyer, J.C., and Okum, D.A., Water and Wastewater Engineering, Vol. 2, John Wiley and Sons, Inc., New York, 1968.
- Flugge, S., ed. Encyclopedia of Physics, Volume III/1, Principles of Classical Mechanics and Field Theory, Springer-Verlag, Berlin, 1960.
- Forrest, J.B., and MacFarlane, I.C., "Field Studies of Response of Peat to Plate Loading," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 95, No. SM4, 1969, pp. 949-967.
- Gershowitz, H., "Rubbish: An Overview," Professional Engineer, July 1971, pp. 35-37.

- Gibson, R.E., England, G.I., and Hussey, M.J.L., "The Theory of One-Dimensional Consolidation of Saturated Clays," Geotechnique, Vol. 17, No. 3, 1967, pp. 261-273.
- Gloor, W.E., and Klug, E.D., Cellulose Data, Interscience Publishers, Inc. New York, 1955.
- Goodman, L.J., and Lee, C.N., "Laboratory and Field Data on Engineering Characteristics of Some Peat Soils," Proceedings of the Eighth Muskeg Research Conference, National Research Council, 1962, pp. 107-128.
- Gotaas, H.B., "Composting-Sanitary Disposal and Reclamation of Organic Waste," World Health Organization, Geneva, 1956.
- Gude, G., "The Solid Waste Problem," Professional Engineer, July 1971, pp. 36-37.
- Hanrahan, E.T., "An Investigation of Some Physical Properties of Peat," Geotechnique, Vol. 4, No. 3, 1954, pp. 108-123.
- Hayes, Williams, Cellulose--The Chemical that Grows, Doubleday and Company, Inc., Garden City, New York, 1963.
- Herman, E.R., and Quon, J.E., Pollution Potential of Compressed Solid Waste, Northwestern University, December 1969.
- Ibrahim, H.A., and Brutsaert, W., "Intermittent Vertical Infiltration into Soils with Hysteresis," Journal of the Hydraulics Division, American Society of Engineers, Vol. 94, No. HY1, 1968, pp. 113-138.
- Institute of Resources Research of Japan, "Decomposition of Compressed Mass of Waste and Water Contamination," 1968.
- Jackson, R.D., Reginato, R.J., and Van Badel, C.H.N., "Comparison of Measured and Calculated Hydraulic Conductivities of Unsaturated Soils," Water Resources Research, Vol. 1, No. 3, 1965, pp. 375-380.
- Jennings, J.E.B., and Burland, J.B., "Limitations to the Use of Effective Stresses in Partly Saturated Soils," Geotechnique, Vol. 13, No. 3, 1963, pp. 132-144.
- Jeris, J.S., and McCarty, P.L., "The Biochemistry of Methane Fermentation Using C^{14} Tracers," Journal of the Water Pollution Control Federation, Vol. 37, No. 2, 1965, pp. 178-192.
- Kenahan, C.B., "Solid Waste Resources Out of Place," Environmental Science and Technology, Vol. 5, No. 7, 1971, pp. 594-600.
- Langfelder, L.J., Chen, C.F., and Justice, J.A., "Air Permeability of Compacted Cohesive Soils," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 94, No. SM4, 1968, pp. 981-1001.

- Lawrence, A.W., and McCarty, P.L., "Kinetics of Methane Fermentations in Anaerobic Treatment," Journal of the Water Pollution Control Federation, Vol. 41, No. 2, Part 2, 1969, pp. R1-R17.
- Lea, N.D., "Notes on the Mechanical Properties of Peat," Proceedings of the Fourth Muskeg Research Conference, National Research Council, 1958, pp. 53-57.
- Lea, N.D., and Brawner, C.O., "Highway Design and Construction over Peat Deposits in Lower British Columbia," Highway Research Board Record, No. 7, Washington, D.C., 1963, pp. 1-33.
- Loehr, R.C., "Design of Anaerobic Digestion Systems," Journal of the Sanitary Engineering Division, American Society of Civil Engineers, Vol. 92, No. SA 1, 1966, pp. 19-30.
- Long, R.R., Mechanics of Solids and Fluids, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1961.
- MacFarlane, I.C., The Consolidation of Peat--A Literature Review, Technical Paper No. 195, National Research Council of Canada, 1965.
- MacFarlane, I.C., Muskeg Engineering Handbook, University of Toronto Press, 1969.
- MacFarlane, I.C., and Radforth, N.W., "A Study of the Physical Behavior of Peat Derivatives under Compression," Proceedings of the Tenth Muskeg Research Conference, National Research Council of Canada, Ottawa, 1965, pp. 159-164.
- Marshall, T.J., "A Relation between Permeability and Size Distribution of Pores," Journal of Soil Science, Vol. 9, No. 1, 1958, pp. 1-8.
- McNabb, A., "A Mathematical Treatment of One-Dimensional Soil Consolidation," Quarterly of Applied Mathematics, Vol. XVII, No. 4, 1960, pp. 337-347.
- Means, R.E., and Parcher, J.V., Physical Properties of Soils, Charles E. Merrill Books, Inc., Columbus, Ohio, 1963.
- Millington, R.J., and Quirk, J.P., "Permeability of Porous Media," Nature, Vol. 183, 1959, pp. 387-388.
- Millington, R.J., and Quirk, J.P., "Permeability of Porous Solids," Transactions of the Faraday Society, Vol. 57, Part 7, 1961, pp. 1200-1207.
- Mitchell, J.K., "Shearing Resistance of Soils as a Rate Process," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 90, No. SM1, 1964, pp. 29-62.
- Mitchell, J.K., Campanella, R.G., and Singh, A., "Soil as a Rate Process," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 94, No. SM1, 1968, pp. 231-253.

- Mitchell, J.K., Hooper, D.R., and Campanella, R.G., "Permeability of Compacted Clay," Journal of the Soil Mechanic and Foundations Division, American Society of Civil Engineers, Vol. 91, No. SM4, 1965, pp. 41-65.
- Moran, Proctor, Maeser, and Rutledge, Study of Deep Soil Stabilization by Vertical Sand Drain, U.S. Department of Navy, Bureau of Yards and Docks, Report No. Y 88812, Washington, D.C., 1958.
- Moskowitz, J., Swelling Properties of Compressed Refuse, M.S. Thesis, Northwestern University, 1969.
- Olson, R.E., and Langfelder, L.J., "Pore Water Treatment in Unsaturated Soils," Journal of the Soil Mechanic and Foundations Division, American Society of Civil Engineers, Vol. 91, No. SM4, 1965, pp. 127-150.
- Peaceman, D.W., "Numerical Solution of the Non-Linear Equations for Two-Phase Flow Through Porous Media," in Non-Linear Partial Differential Equations, ed. by W. F. Ames, Academic Press, New York, 1967, pp. 171-191.
- Polubarinova-Kochina, P.Y.A., translated by J. M. Roger de Wiest, Theory of Ground Water Movement, Princeton University Press, Princeton, N.J., 1962.
- Poskitt, T.J., "The Consolidation of Saturated Clay with Variable Permeability and Compressibility," Geotechnique, Vol. 19, No. 2, 1969, pp. 234-252.
- Poskitt, T.J., "Consolidation of Clay and Peat with Variable Properties," Journal of the Soil Mechanic and Foundations Division, American Society of Civil Engineers, Vol. 97, No. SM6, 1971, pp. 841-880.
- Raats, P.A.C., and Klute, A., "Transport Soils: The Balance of Mass," Soil Science Society of America Proceedings, Vol. 32, No. 2, 1968a, pp. 161-166.
- Raats, P.A.C., and Klute, A., "Transport in Soils: The Balance of Momentum," Soil Science Society of America Proceedings, Vol. 32, No. 4, 1968b, pp. 452-456.
- Raats, P.A.C., and Klute, A., "One-Dimensional, Simultaneous Motion of the Aqueous Phase and the Solid Phase of Saturated and Partially Saturated Porous Media," Soil Science, Vol. 107, No. 5, 1969, pp. 329-333.
- Ralph Stone, Inc., "Landfill Compaction Studies," First Annual Report, Los Angeles, September 30, 1967.
- Ralph Stone, Inc., "Solid Waste Landfill Stabilization," Second Annual Report, Los Angeles, September 30, 1968.
- Raymond, G.P., "Laboratory Consolidation of Some Normally Consolidated Soils," Canadian Geotechnical Journal, Vol. 3, No. 4, 1966, pp. 217-234.

- Raymond, G.P., "Consolidation of Deep Deposits of Homogeneous Clay," Geotechnique, Vol. 19, No. 4, 1969, pp. 478-494.
- Reinhardt, J., Project Director, "A Report on the Demonstration of the Gondard Grinding Mill for Pulverizing Refuse and Landfilling Refuse without Daily Cover," Department of Health, Education and Welfare, Grant No. S-D01-ul-00004, by the City of Madison and University of Wisconsin, 1971.
- Rode, A.A., Soil Science, Translated from Russian, published for the National Science Foundation by the Israel Program for Scientific Translation, Jerusalem, 1962.
- Schiffman, R.L., "Consolidation of Soil under Time-Dependent Loading and Varying Permeability," Proceedings of the Highway Research Board, Vol. 37, 1958, pp. 584-617.
- Schroeder, E.D., and Busch, A.W., "Mass and Energy Relationships in Anaerobic Digestion," Journal of the Sanitary Engineering Division, American Society of Civil Engineers, Vol. 92, No. SA1, 1966, pp. 85-97.
- Schroeder, J., and Wilson, N.E., "The Analysis of Secondary Consolidation of Peat," Proceedings of the Eighth Muskeg Research Conference, National Research Council of Canada, Ottawa, 1962, pp. 130-136.
- Schuurman, I.E., "The Compressibility of an Air/Water Mixture and a Theoretical Relation Between the Air and Water Pressures," Geotechnique, Vol. 16, No. 4, 1966, pp. 269-281.
- Singh, A., and Mitchell, J.K., "General Stress-Strain-Time Function for Soils," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 94, No. SM1, 1968, pp. 21-46.
- Skempton, A.W., "The Pore Pressure Coefficients A and B," Geotechnique, Vol. 4, No. 4, 1954, pp. 143-147.
- Soo, S.L., Fluid Dynamics of Multiphase Systems, Blaisdell Publishing Company, Waltham, Massachusetts, 1967.
- Technical Practice Committee--Subcommittee on Sludge Digestion, "Anaerobic Sludge Digestion--MOP 16," Journal of the Water Pollution Control Federation, Vol. 38, No. 10, 1966, pp. 1683-1702.
- Terzaghi, K., and Peck, R.B., Soil Mechanics in Engineering Practice, Second ed., John Wiley and Sons, Inc., New York, 1967.
- Tessier, G., "Two Examples -- Types of Road Construction on Muskegs in Quebec," Proceedings of the Eleventh Muskeg Research Conference, Ottawa, 1966, pp. 92-141.
- Thadani, N., "Solid Waste Collection Systems," Professional Engineer, July 1971, pp. 36-39.

- Toriyama, K., and Sawada, T., "On the Consolidation of Partly Saturated Soils Compacted Wet of Optimum Moisture Content," Soils and Foundations, Vol. 8, No. 3, 1968, pp. 63-86.
- Wahls, H.E., "Analysis of Primary and Secondary Consolidation," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 88, No. SM6, 1962, pp. 207-231.
- Walker, L.K., "Secondary Compression in the Shear of Clays," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 95, No. SM1, 1969, pp. 167-188.
- Walker, L.K., Discussion, Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 95, No. SM1, 1969, pp. 409-414.
- Walker, L.K., "Undrained Creep in a Sensitive Clay," Geotechnique, Vol. 19, No. 4, 1969, pp. 515-529.
- Wilson, N.E., Radforth, N.W., MacFarlane, I.C., and Lo, M.B., "The Rates of Consolidation for Peat," Proceedings of the Sixth International Conference on Soil Mechanics and Foundation Engineering, Montreal, Vol. I, 1965, pp. 407-411.
- Wisler, C.O., and Brater, E.F., Hydrology, Second ed., John Wiley and Sons, Inc., New York, 1959.
- Wu, T.H., Resendiz, D., and Neukircher, R.J., "Analysis of Consolidation by Rate Process Theory," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 92, No. 6, 1966, pp. 229-250.
- Yong, R.N., and Warkentin, B.P., Introduction to Soil Behavior, The Macmillan Company, New York, 1965.
- Yoshimi, Y., and Osterberg, J.O., "Compression of Partially Saturated Cohesive Soils," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 89, No. SM4, 1963, pp. 1-24.
- Zaslavsky, D., "Saturated and Unsaturated Flow Equation in an Unstable Porous Medium," Soil Science, Vol. 98, No. 5, 1964, pp. 317-321.

APPENDIX A

METHOD OF SOLUTION OF SIMULTANEOUS EQUATIONS

The method used to solve the two simultaneous equations is modified from one proposed by Barden (1965a). In their most general form, the two simultaneous equations are

$$\begin{aligned} \frac{\partial \lambda}{\partial \tau} = & S_e^N \left(\frac{e}{e_0} \right)^N \left(\frac{1+e}{1+e_0} \right)^2 \left(\frac{\partial^2 u}{\partial z^2} + A' \left(\frac{\partial u}{\partial z} \right)^2 \right) \\ & + \frac{\partial u}{\partial \tau} \frac{H^2}{K_0(1+e)} \frac{(1+e)g}{S} \psi - \frac{e}{S} \frac{\partial S}{\partial \tau} \end{aligned} \quad \begin{array}{l} \text{(A-1)} \\ \text{(8-3)} \end{array}$$

and

$$\frac{\partial \lambda}{\partial \tau} = \frac{1}{\Delta p \phi_c \tau_R} \sinh(\phi_c \Delta p (\lambda - u)) \quad \begin{array}{l} \text{(A-2)} \\ \text{(6-16)} \end{array}$$

where λ , S_e , e and u are variables. The variables S_e and e are calculated independently of the two above equations using the relationships

$$S_{t+\Delta t} = \frac{S_t}{1 + \left(\frac{P_{ft}}{P_{ft+\Delta t}} + 1 \right) (1 - S_t + h_c S_t)} \quad \begin{array}{l} \text{(A-3)} \\ \text{(5-49)} \end{array}$$

and

$$e_{t+\Delta t} = e_f + \lambda_t (e_0 - e_f) \quad \text{(A-4)}$$

The partial differential terms in equations A-1 and A-2 are approximated using the general finite difference approximations. These approximations are:

For the variation of λ with time:

$$\frac{\partial \lambda}{\partial T} = \frac{\lambda_{(T+1, Z)} - \lambda_{(T, Z)}}{\Delta T} \quad (A-5)$$

for fluid pressure variations with location:

$$\frac{\partial^2 U}{\partial Z^2} = \frac{U_{(T, Z+1)} - 2U_{(T, Z)} + U_{(T, Z-1)}}{\Delta Z^2} \quad (A-6)$$

and

$$\frac{\partial U}{\partial Z} = \frac{U_{(T, Z+1)} - U_{(T, Z)}}{\Delta Z} \quad (A-7)$$

for saturation variation with time

$$\frac{\partial S}{\partial T} = \frac{S_{(T+1, Z)} - S_{(T, Z)}}{\Delta T} \quad (A-8)$$

Due to the symmetry at the mid-plane of the sample, the values of the fluid pressure at the first nodes on each side of the mid-plane are equal and equation A-6 becomes

$$\frac{\partial^2 U}{\partial Z^2} = \frac{2U_{(T, 1)} - 2U_{(T, 0)}}{\Delta Z^2} \quad (A-9)$$

The process used for the solution of the two equations is a simple explicit finite-difference scheme, combined with relaxation techniques to satisfy boundary conditions. The time-space grid used for the study is shown in Figure A-1.

The flow chart for the computer solution of the two equations is shown in Figure A-2, and the steps involved are discussed in the following paragraphs. The first step in the process is to establish the boundary and initial conditions for time zero. Next, the time is increased by a predetermined amount, and the instantaneous values of the void ratio, saturation and variable coefficients are calculated. Using equation A-1, the values of the settlement at the different nodes are found.

The fluid pressure for each node is calculated by equating the right hand sides of equations A-1 and A-2 and solving for the pressures. The resulting pore fluid function is

$$\begin{aligned}
 u_{(T+1, Z+1)} = & 2 u_{(T+1, Z)} - u_{(T+1, Z-1)} \\
 & + \frac{1}{\Delta Z^2} \left[\frac{\sinh(\alpha_c \Delta p (\lambda_{(T+1, Z)} - u_{(T+1, Z)}))}{\alpha_c \Delta p T_R S_e^N \left(\frac{e}{e_0} \right)^N \left(\frac{1+e_0}{1+e} \right)^2} - A^* (u_{(T+1, Z)} - u_{(T+1, Z-1)})^2 \right] \\
 & - \frac{1}{\Delta Z^2} \left[\frac{1}{S_e^N \left(\frac{e}{e_0} \right)^N \left(\frac{1+e_0}{1+e} \right)} \left(\gamma^* - \frac{e}{S} \frac{\partial S}{\partial T} \right) \right]
 \end{aligned}
 \tag{A-10}$$

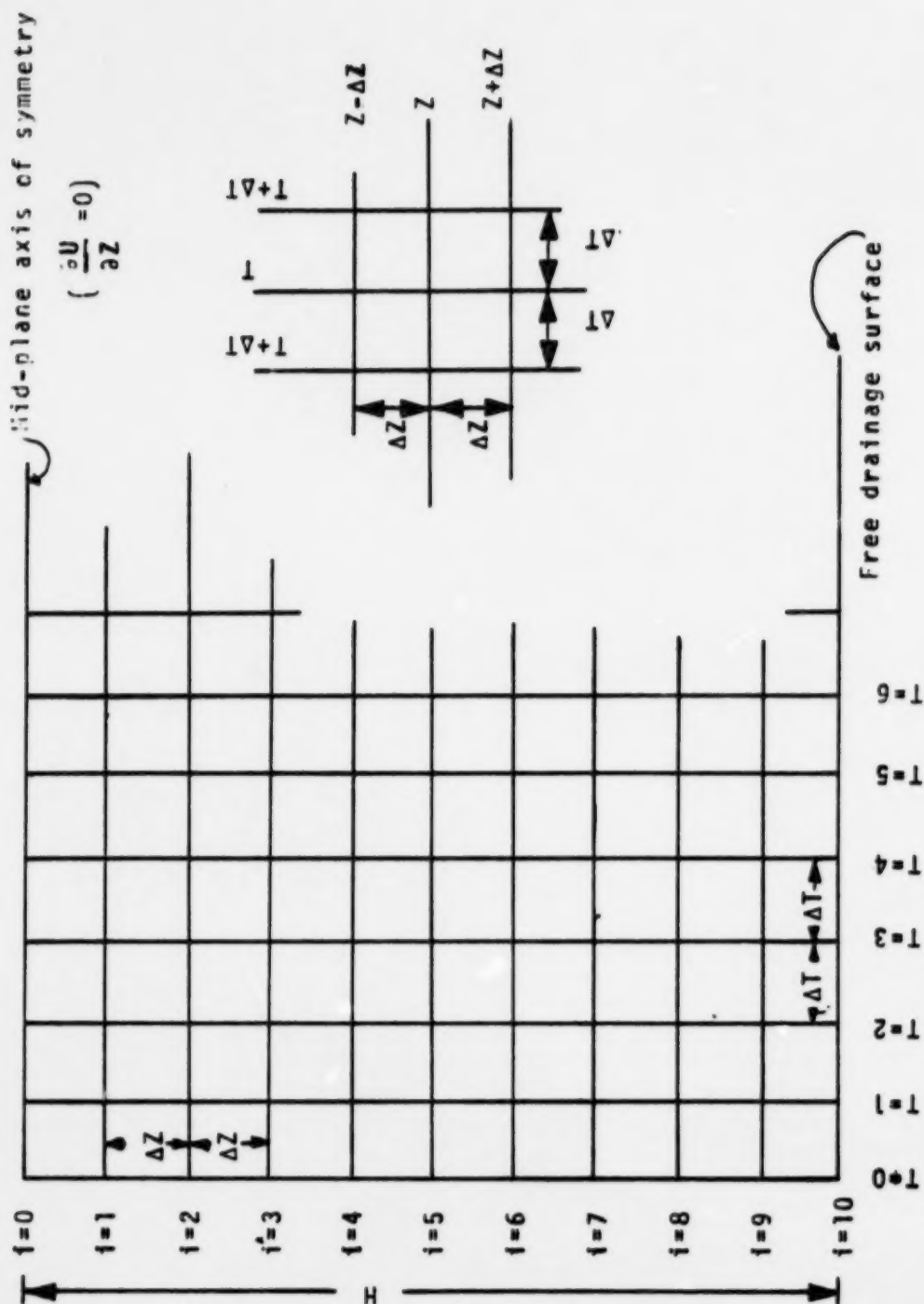


Figure A-1: Time -space grid.

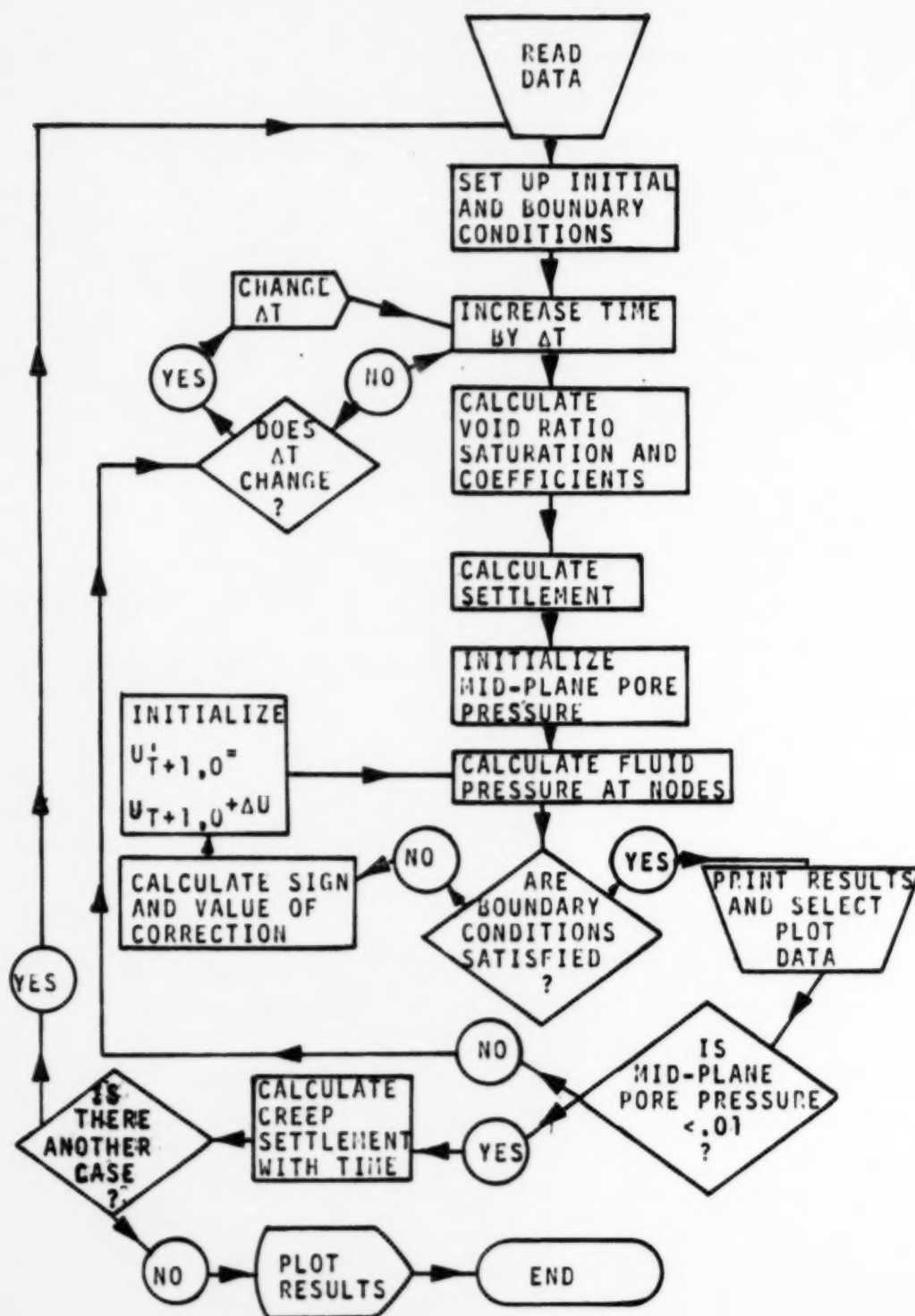


Figure A-2: Flow chart for computer program.

The values at the space locations are then calculated using the previous value of the fluid pressure at the mid-plane as the first approximation to the new mid-plane fluid pressure. After the first iteration, the pressure calculated at the boundary node is greater than that dictated by the boundary conditions, and the process is repeated using a slightly smaller mid-plane pressure as the starting point. The process is continued until either the difference between the computed and known values of the boundary pressure is within acceptable limits, or the computed value is negative. If a negative result is produced, the mid-plane pressure is increased by $3/8$ ths the amount that it was previously reduced. This continues until either another sign change occurs, or the calculated pressure value is satisfactory. The iterations continue until the boundary value is reached or an excessive number of iterations are made. The number of iterations vary greatly from problem to problem and during the problem itself. In general, as Barden found (1965a), the closer to the classical type behavior, the more iterations. As a consolidation case progresses with time, the amount of fluid pressure dissipated per unit time decreases, and the number of iterations decrease.

When the new fluid pressure values have been determined, the time is incremented, new settlement values are calculated and then the iteration process for the pressures is repeated. The process is continued until the mid-plane pore pressure is less than one one-hundredth of its original value. When this occurs the remaining settlement is assumed to be due only to the structural creep and is found using equation A-2 alone.

Barden discusses the stability and convergence of his system and, in general, the same observations were made in this study. Several different

values of ΔT and Δx are tried in this study and except for extreme cases where the creep effects are quite small, the values of $\Delta x = 0.1$ and $\Delta T = 0.001$ proved quite satisfactory. When the creep influence is considerable, the time increment is increased by as much as tenfold after a specified number of time increments have occurred.

APPENDIX B

STABILITY AND CONVERGENCE

When the model equations are solved separately or simultaneously, the stability and convergence of the numerical system must be considered. There are no general rules governing the behavior of non-linear equations, but according to Douglas (1960) and Peaceman (1967) the convergence or divergence of a difference scheme will not be affected by the presence of the lower order terms. Thus, with the systems at hand the study can concentrate on the behavior of the higher order terms. A Von Neuman stability analysis for the fluid dissipation equation is presented below; subsequently the behavior of this equation alone and then that of the combined model is discussed.

The equation for the dissipation of the pore fluid is of the form

$$\frac{\partial u}{\partial \tau} = K_1 \left(\frac{\partial^2 u}{\partial z^2} \right) + K_2 \left(\frac{\partial u}{\partial z} \right)^2 + K_3 \quad (B-1)$$

Since the lower order term is bounded and should not affect the convergence of the system, and since K_2 and K_3 are also bounded, we have

$$K_2 \left(\frac{\partial u}{\partial z} \right)^2 + K_3 \leq M \quad (B-2)$$

where M is some value such that

$$|M| \leq C \quad (B-3)$$

where C is some finite, positive constant. If a new variable Y is introduced

so that

$$Y = u + \frac{x^2 M}{2K_1} \quad (B-4)$$

or

$$u = Y - \frac{x^2 M}{2K_1} \quad (B-5)$$

it follows then that

$$\frac{\partial u}{\partial x} = \frac{\partial Y}{\partial x} - \frac{XM}{K_1} \quad (B-6)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 Y}{\partial x^2} - \frac{M}{K_1} \quad (B-7)$$

$$\frac{\partial u}{\partial \tau} = \frac{\partial Y}{\partial \tau} \quad (B-8)$$

Substitution of equations B-6, B-7, and B-8 into B-1 yields

$$\frac{\partial Y}{\partial \tau} = K_1 \left(\frac{\partial^2 Y}{\partial x^2} \right) \quad (B-9)$$

Rewriting this equation using a simpler forward difference scheme yields

$$\frac{Y_{(i,j+1)} - Y_{(i,j)}}{\Delta \tau} = K_1 \frac{Y_{(i+1,j)} - 2Y_{(i,j)} + Y_{(i-1,j)}}{\Delta x^2} \quad (B-10)$$

Now assume that

$$y = e^{\alpha t} e^{iBx} \quad (B-11)$$

and substitute it into equation B-10, producing

$$\begin{aligned} & \frac{e^{(t+\Delta t)} e^{iBx} - e^{\alpha t} e^{iBx}}{\Delta x^2} \\ &= K_1 \frac{e^{\alpha t} (e^{iB(x+\Delta x)} - 2e^{iBx} + e^{iB(x-\Delta x)})}{\Delta x^2} \end{aligned} \quad (B-12)$$

or

$$e^{\alpha \Delta t} - 1 = \frac{\Delta T}{\Delta x^2} K_1 (e^{iB\Delta x} - 2 + e^{-iB\Delta x}) \quad (B-13)$$

$$= \frac{\Delta T}{\Delta x^2} K_1 [2 (\cos (B\Delta x) - 1)] \quad (B-14)$$

$$= - \frac{\Delta T}{\Delta x^2} 4 \sin^2 \left(\frac{B\Delta x}{2} \right) \quad (B-15)$$

For stability to be guaranteed, we need

$$|e^{\alpha \Delta t}| \leq 1 \quad (B-16)$$

or

$$\left| 1 - K_1 \frac{\Delta T}{\Delta x^2} \sin^2 \left(\frac{B\Delta x}{2} \right) \right| \leq 1 \quad (B-17)$$

We always have

$$0 \leq \sin^2 \left(\frac{B \Delta x}{2} \right) \leq 1 \quad . \quad (B-18)$$

Therefore we need

$$0 \leq 4 K_1 \frac{\Delta T}{\Delta x^2} \leq 2 \quad (B-19)$$

or

$$0 \leq K \frac{\Delta T}{\Delta x^2} \leq \frac{1}{2} \quad . \quad (B-20)$$

Thus according to this analysis, the solution should be stable as long as the conditions of equation B-20 are satisfied. Convergence of the system will be established by taking smaller and smaller values of the ratio by varying both Δx and ΔT . Carnahan, Luther and Wilkes (1969) show that for a non-linear equation of the same form as equation B-1, the value of $\Delta x / \Delta T$ must not be kept constant when evaluating the convergence. The evaluation of the convergence of the system was therefore carried out using dimensionless time values of .01, .001, .0005 and .0001 and grid spacings of .2, .1, .05, and .025 to produce $\Delta T / \Delta x^2$ values of 0.25, 0.20, 0.1, 0.05, 0.04, 0.025, and 0.01.

The finite difference solution to the fluid pressure dissipation equation was found to be stable for all values of the ratio $\Delta T / \Delta x^2$. However, when $\Delta T / \Delta x^2 = .25$, the solutions did not always converge to those obtained for lower ratios. For economical use of the computer, the value of ΔT or

the number of time increments required to reach a specific dimensionless time is critical. Where a time increment value of 0.01 was used, convergence was not always obtained. The other three values used were generally satisfactory. The solutions for this equation (no. 6-10) are found to agree within one percent (1%) for all $\Delta T/\Delta x^2 = 0.1$ and in most cases for $\Delta T/\Delta x^2 \leq .20$, and therefore a time-space grid of $\Delta T = .001$ and $\Delta x = .1$ was used for the analysis. No stability problems were encountered when K_3 included the effects of gas production or changing saturation, but if the term is great enough it may dominate the model's behavior. Several variations in the ratio $\Delta T/\Delta x^2$ gave the same results when the additional effects are considered and it is assumed that the given grid will produce a convergent solution.

Stability and convergence of the results produced from the simultaneous solution of the two equations may depend upon, in addition to the grid spacing, the magnitude of the difference between the actual and the calculated pressure at the boundary, and the relative influence of each of the equations. The same values of ΔT and Δx were used to check on the requirements for grid spacing, and except when there is little influence of the creep function, the values of $\Delta x = .1$, $\Delta T = .001$ are satisfactory.

The number of iterations required to satisfy the boundary conditions for the pore pressure calculations depends upon both the accuracy required of the solution and the importance of each mode of settlement. The accuracy of the solution is taken to be the closeness to which the calculated pore pressure at the free boundary approximates the known value at the boundary node. Boundary error values of 10^{-3} , 10^{-4} , 10^{-5} , 10^{-6} , 10^{-7} , 10^{-9} , and 10^{-12} were used and interesting results were obtained. As would be

expected, the lower acceptance value, 10^{-3} , produced enough inaccuracy in the fluid pressure values at the nodes to generate instability early in the calculations. When the acceptance requirement is 10^{-4} , the instability never occurs before a dimensionless time of unity, with most solutions being stable. A value of 10^{-5} produces stable and convergent results. As the accuracy of the fluid pressure at the boundary is increased to acceptance values of 10^{-6} , 10^{-7} , 10^{-9} and 10^{-12} , the number of iterations required to produce a satisfactory approximation to the pressure curve increases accordingly. This increase in the number of calculations produces a more refined pressure curve, but does not improve the stability of any case. Thus, when the time increment is .001 an acceptance value of 10^{-5} is sufficient. In extremely stable cases, a time increment of .005 and .01 can be used in the later stages of calculations, and the acceptance value is reduced to 10^{-7} .

As Barden (1965a) states, the solution to the simultaneous equations breaks down when the influence of the structural resistance equation is small. When the value of $\Delta p \phi_c$ is greater than 30, the simultaneous solution breaks down and becomes unstable after a time value .2 to .5. If the value of ΔT is decreased, the period of stability may be lengthened by 25 to 30 percent. When $\Delta p \phi_c$ lies between 20 and 30, the solution is stable and convergent to a dimensionless time of 1.0 and 1.5. However, because all results are bounded by the classical and $\Delta p \phi_c = 15$ solutions, approximate solutions for the intermediate cases are easily obtained. At the other extreme, when $\Delta p \phi_c = 1$, the system of equations is very stable and convergence is easily obtained.

In the cases when $\Delta p \phi_c \leq 10$, the soundness of the system is so

evident that after $T = 0.5$, the time increment is increased by fivefold to .005. Then when $T = 1.0$, the value of ΔT is again increased to .01. This results in a much shorter calculation time and solutions which agree within 1 percent with those where no change in ΔT occurs.